Empirical Estimation of Probability Distribution of Extreme Responses of Turret Moored FPSOs

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ABSTRACT

In this article, the probability distribution of dynamic responses of turret moored FPSOs is closely studied and the effects of non-linearity on the response distribution and the extreme statistics are evaluated. For this purpose, sample data sets obtained from two experimental model tests studying the response of typical external turret mooring systems designed for deepwater and shallow-water conditions are utilized. The focus here is on the extreme statistics of the mooring line tension and vessel horizontal offset. The probability distribution of measured data are estimated using commonly used distribution models of linear and non-linear random variables. Additionally, the application of three-parameter Rayleigh distribution model in estimating the probability distribution of linear and non-linear responses of turret-moored FPSOs is studied. The performance of these distribution models in representing the sample distribution and predicting extreme statistics is evaluated.

KEY WORDS: Non-linear response, Turret moored system, Extreme statistics, Probability distribution estimation, three-parameter Rayleigh

INTRODUCTION

Some of the dynamic responses of turret moored Floating Production, Storage, and Offloading (FPSO) units in extreme environments are known to be non-linear random variables. Among those are the horizontal offset of the FPSO and the tension in the mooring legs. The non-linearity in these variables depends on many factors including the mooring system design, environmental conditions, the vessel characteristics, etc. In analysis of these turret moored FPSOs, complex analytical, numerical, and experimental methods are utilized to model the non-linear responses. Additionally, statistical models are applied to predict the probability distribution of the random responses. The probability distributions are eventually used to predict the extreme statistics, e.g. expected maximum and most probable maximum, and to obtain the design values.

It is common in the field of offshore engineering to assume that the amplitudes of a random process follow the Rayleigh distribution law. The design guidelines and recommendations usually use the Rayleigh model to estimate the extreme statistics of mooring line tension and vessel offset (see e.g. API 2005, ABS 2013, and DNV 2010). The Rayleigh distribution model assumes that the random process can be approximated as a linear random variable. This assumption may result in significant underestimation of extreme statistics when the response is not linear. Previous studies on the slow-drift response of floating structures in irregular seas indicate that these random variables could be highly non-linear (e.g. Naess 1986, Stansberg 1991, Stansberg 1992, Stansberg 2000). Liu and Bergdahl (1998) show that the probability distribution of extreme mooring line tensions caused by wave-frequency excitations may also deviate from the Rayleigh distribution of the linear random variables.

In this study, the probability distribution of mooring line tension and vessel horizontal offset are closely studied. For this purpose, experimental data sets obtained from two model tests studying the response of external turret FPSOs in extreme environmental conditions are used. The cases studied here represent typical deepwater and shallow-water mooring system designs. The probability distribution of low-frequency, wave frequency, and total mooring line tension as well as the low-frequency horizontal offset is estimated using commonly used probability distribution models. As an alternative, the three-parameter Rayleigh distribution model is applied to estimate the probability distribution of turret moored reponses. The three-parameter Rayleigh distribution model was originally developed by Izadparast and Niedzwecki (2009, 2010) for second-order Stokes type variables. Here, the performance of this model in capturing the probability distribution of low-frequency and wave-frequency responses is evaluated.

PROBABILITY DISTRIBUTION OF NON-LINEAR AMPLITUDES

The amplitudes of a random variable η are defined as the maximum observation between each two consecutive zero-upcrossings. The normalized form of amplitudes is obtained from

$$\zeta = \frac{a-\mu_\eta}{\sigma_\eta} \qquad (1)$$

where μ_η is the mean η, σ_η is the standard deviation of η, and a is the amplitude. Assuming that η is a linear narrow-banded random variable, it can be shown that the amplitudes ζ follow a Rayleigh distribution law with cumulative distribution function (CDF) of

(Longuet-Higgins 1952)

$$F_{\zeta}\left(x\right)=1-\exp\left(-x^{2}/2\right) \tag{2}$$

Rayleigh distribution has been widely used for ocean engineering applications to predict the extreme statistics. However, it is well known that the distribution of non-linear random variables deviates from Rayleigh distribution. This issue is more sensible on the tail of the distribution where the non-linearity has a larger contribution. A simplified representation of the second-order random variables can be obtained by assuming that the non-linear term is closely related to the squared of the linear variable, specifically

$$\zeta_n=\frac{\zeta^2}{2} \tag{3}$$

where ζ_n is the amplitude of the non-linear process. Using this transformation and the distribution of linear amplitudes Eq. (2), the CDF of ζ_n becomes

$$F_{\zeta_n}\left(x\right)=1-\exp\left(-x\right) \tag{4}$$

which is the well-known Exponential distribution. The Exponential probability distribution is commonly used to describe the tail distribution of second-order random variables. A more general form of the Exponential distribution was introduced by Stansberg (1991) for estimation of extreme values of non-linear slow-drift responses. The model was used by Fylling and Stansberg (1992) and Stansberg (1992) for extreme offsets and anchor line loads of turret moored systems and reasonable agreement between the model predictions and experimental data was observed. Later on, Stansberg (2000) updated his Exponential model to improve its performance for systems with very high and very low low-frequency damping. In this model the non-linear amplitudes are defined as

$$\zeta_n=A\left(\frac{\zeta^2}{2}+B\right) \tag{5}$$

and consequently the CDF of the Exponential distribution is changed into

$$F_{\zeta_n}\left(x\right)=1-\exp\left(-\left(\frac{x}{A}-B\right)\right) \tag{6}$$

The three-parameter Weibull distribution is another distribution model that has been widely used to estimate the probability distribution of non-linear amplitudes. The structural form of the Weibull distribution is defined by three parameters, i.e. scale λ, shape κ, and location ρ parameters, specifically

$$F_{\zeta_n}\left(x\right)=1-\exp\left(-\left(\frac{\left(x-\rho\right)}{\lambda}\right)^{\kappa}\right) \tag{7}$$

The Weibull distribution is a more general form of the Rayleigh distribution and assumes the following relation between the linear and non-linear random amplitudes.

$$\zeta_n=\frac{\lambda}{2^{1/\kappa}}\zeta^{2/\kappa}+\rho \tag{8}$$

A Weibull distribution with $\kappa=2$, $\lambda=\sqrt{2}$, and $\rho=0$ reduces to the Rayleigh distribution of linear amplitudes. The three-parameter Weibull distribution model is usually used as a powerful data analysis tool but the model parameters do not have clear physical interpretations.

Izadparast and Niedzwecki (2009, 2010) introduced the three-parameter Rayleigh distribution model for the amplitudes of second-order Stokes type random variables. In this model, the non-linear random variable is defined using the quadratic transformation

$$\zeta_n=\alpha\zeta+\beta\zeta^2+\gamma \tag{9}$$

where α is the amplification of the linear term, β is the amplification of the quadratic term, and γ is the shifting between linear and non-linear variables. The three-parameter Rayleigh distribution essentially combines the contribution of Rayleigh and Exponential distributions. As shown in previous studies (Stansberg 1991, 1992, and 2000 and Fylling and Stansberg 1992) the probability distribution of slow-drift response is usually between the bounds defined by Rayleigh distribution and Exponential distribution and therefore the three-parameter Rayleigh distribution should be able to model those behaviors. The three-parameter Rayleigh model assumes that the linear and non-linear terms are phased-locked and their peaks happen at the same time; therefore, the three-parameter Rayleigh model is appropriate for representing the probability distribution of large amplitudes. Applying the random variable transformation rule on Eq. (9), the CDF of the three-parameter Rayleigh model for $\beta>0$ is obtained as

$$F_{\zeta_n}\left(x\right)=1-\exp\left(-\frac{\left(\chi-\alpha\right)^2}{8\beta^2}\right) \tag{10}$$

where,

$$\chi=\left(\alpha^2+4\beta\left(x-\gamma\right)\right)^{1/2} \tag{11}$$

In the case of $\beta<0$, the CDF becomes

$$F_{\zeta_n}\left(x\right)=\left[1-\exp\left(-\frac{\left(\chi-\alpha\right)^2}{8\beta^2}\right)\right]H_{\gamma}\left(x\right)+\exp\left(-\frac{\left(\chi+\alpha\right)^2}{8\beta^2}\right) \tag{12}$$

where $H_{\gamma}\left(x\right)$ is the step function and has a value of unity for $x\geq\gamma$ and is zero for $x<\gamma$.

EXTREME STATISTICS

The CDF of the maxima $\zeta_{\max}$ in N independent and identically distributed events can be obtained from the ordered value statistics theory (Leadbetter, Lindgren, and Rootzen 1983), specifically

$$F_{\zeta_{\max}}\left(x\right)=\left[F_{\zeta_n}\left(x\right)\right]^N \tag{13}$$

From that, the expected maximum $E\left(\zeta_{\max}\right)$ can be estimated from the integration

$$E\left(\zeta_{\max}\right)=\int_{-\infty}^{+\infty}x\,dF_{\zeta_{\max}}\left(x\right) \tag{14}$$

It can be shown that for large N, all the distribution models introduced in the previous section belong to the Gumbel maximal domain of attraction with asymptotic distribution of

$$F_{\zeta_{\max}}\left(x\right)=\exp\left(-\exp\left(-\left(x-a_N\right)/b_N\right)\right) \tag{15}$$

where a_N and b_N are the Gumbel distribution parameters. The Gumbel distribution parameters are related to the parameters of the three-parameter Rayleigh distribution model as

$$\begin{aligned} a_N&=\gamma+2\beta\ln\left(N\right)+\alpha\left(2\ln\left(N\right)\right)^{1/2}\\ b_N&=2\beta+\alpha\left(2\ln\left(N\right)\right)^{-1/2}\end{aligned} \tag{16}$$

The relation between the Gumbel distribution parameters and the parameters of the Weibull distribution is obtained in the form of

$$\begin{aligned} a_N&=\rho+\lambda\left(2\ln\left(N\right)\right)^{1/\kappa}\\ b_N&=\lambda\left[\left(\ln\left(N\right)+1\right)^{1/\kappa}-\left(\ln\left(N\right)\right)^{1/\kappa}\right]\end{aligned} \tag{17}$$

The estimates of a_N and b_N for the other distributions can be obtained by making the following substitutions in Eq. (16)

- For Rayleigh distribution Eq. (2): $\beta=0$, $\gamma=0$, and $\alpha=1$
- For Exponential distribution Eq. (3): $\beta=1/2$, $\gamma=0$, and $\alpha=0$

- For Stansberg's Exponential distribution Eq. (5): $\beta = A/2$, $\gamma = AB$

Assuming that $\zeta_{\max}$ follows the Gumbel probability distribution function, the expected maximum can be estimated from

$$E(\zeta_{\max}) = a_N + b_N \gamma_{EM} \tag{18}$$

where $\gamma_{EM} \approx 0.5772$ is the Euler-Mascheroni constant. As examples, the normalized expected maximum of the Rayleigh and Exponential distributions in $N = 1000$ events are $E(\zeta_{\max}) = 3.87$ and $E(\zeta_{\max}) = 6.91$, respectively.

Another issue to be discussed is the number of independent cycles N in a storm with a certain duration (e.g. 3hr storm). The design guidelines (e.g. API 2005, ABS 2013, and DNV 2010) recommend estimating the number of cycles as

$$N = T_{storm}/T_z \tag{19}$$

where T_{storm} is the storm duration and T_z is the average mean-upcrossing period of the process. The mean-upcrossing period of a wave-frequency process is in order of 10-20sec, which results in 1080-540 cycles in a 3hr storm. The period of a low-frequency process is commonly in order of 100-300sec and the number of cycles in a 3hr storm is about 108-36. For a true narrow-banded process T_z can be considered as the correlation time of the process and the correlation between the consecutive amplitudes is insignificant. In an actual process with a spectrum of finite width, the consecutive amplitudes are correlated and the number of independent cycles in a signal is different from the number of cycles estimated by Eq. 19. It is common to assume that the wave-frequency response is narrow-banded and use the total number of cycles for extreme estimation. This approximation usually results in slight overestimation of extreme statistics.

Application of Eq. 19 for low-frequency responses remains questionable as these processes are usually not narrow-banded. To address this issue, Naess (1989) estimated the number of statistically independent cycles in slow-drift responses as a function of the low-frequency damping in the system. Stansberg (2000) estimated the number of independent cycles in a low-frequency response by substituting T_z in Eq. 19 with the correlation time of the signal τ defined as

$$\tau = 1/2\omega \tag{20}$$

where ω is the bandwidth of the spectrum calculated from

$$\omega = \left[\int_0^\infty S(f)\,df\right]^2 \Big/ \left[\int_0^\infty \left[S(f)\right]^2 df\right] \tag{21}$$

$S(f)$ is the one-sided spectrum, and f is the frequency. This approach is followed here to estimate the number of independent cycles in low-frequency signals.

Estimating the number of independent cycles in a signal of combined wave-frequency and low-frequency components is even more challenging. Here, the number of observed cycles are used for N which is not theoretically justified. Defining a better estimate for the number of cycles of combined low and wave frequency components requires further studies and is not in the scope of this article.

MODEL PARAMETERS

In order to use the Rayleigh distribution of the linear amplitudes Eq. (2) and the Exponential distribution Eq. (3), one needs the estimates of the mean and standard deviation of the random process η. This makes these two models very attractive when only limited information about the random variable is available. Stansberg's exponential model requires some information about the characteristics of the spectrum of the non-linear response and the input waves. It is worth mentioning that the parameters of the Rayleigh distribution model, Exponential distribution, and Stansberg's Exponential model can be estimated from frequency domain analysis of the system while in order to estimate the parameters of the three-parameter Weibull distribution model and the three-parameter Rayleigh distribution model, a sample timeseries is required.

Estimates of Stansberg's model parameters, i.e. A and B, can be obtained from (Stansberg 2000)

$$\begin{aligned} A &= \left\{(1/A_0) - \left[2s(M-1)^2 \big/ \left(3y_0^{\,3}\right)\right]\right\} \\ B &= D - \left(\sqrt{M}\right)\big/ A \end{aligned} \tag{22}$$

where, M is the ratio of the bandwidth of the wave group spectrum and the bandwidth of the response spectrum,

$$M = \omega_{wave\ group} / \omega_{response} \tag{23}$$

The spectrum bandwidth is estimated from Eq. 21 and the other parameters used in Eq. 22 are estimated from

$$\begin{aligned} s &= \left[(M-1)/(M+1)\right] \\ y_0 &= 4 + \sqrt{M} + A_0 \\ A_0 &= 2\sqrt{M}/(M+1) \\ D &= D_0 - (s/3)\left[(M-1)/y_0\right]^2 \left[(3y_0 - 2A_0)/y_0\right] \\ D_0 &\approx s(0.3M - 0.5) + \left(\sqrt{M}\right)\big/ A_0 \end{aligned} \tag{24}$$

The parameters of the three-parameter Weibull and the three-parameter Rayleigh distribution are estimated using a sample set of amplitudes. There are numerous parameter estimation methods that can be used to estimate the parameters of a probability distribution, e.g. method of maximum likelihood, method of least squares, method of moments, etc. Izadparast and Niedzwecki (2012) used two moment based parameter estimation methods, i.e. conventional method of moments and method of linear moments (L-moments) to estimate the parameters of three-parameter Rayleigh distribution. It was shown in their study that the performance of the two parameter estimation methods are similar for large samples while method of L-moments is more robust for small sample sizes. Here, the method of L-moments is applied to estimate the model parameters of the three-parameter Weibull and the three-parameter Rayleigh distributions. In method of L-moments, the estimates of the parameters are obtained by equating the distribution moments with their corresponding unbiased sample L-moments. This will give a system of equations to be solved for the unknown parameters.

It can be shown that the relations between the first three sample L-moments, i.e. l_1, l_2, and l_3 and the parameters of three-parameter Rayleigh model are,

$$\begin{aligned} \alpha &= 4.1394\left(l_2(\zeta_n) - 3\,l_3(\zeta_n)\right) \\ \beta &= \left(l_2(\zeta_n) - \left(2^{1/2} - 1\right)\alpha\,\Gamma(3/2)\right) \\ \gamma &= l_1(\zeta_n) - 2\beta - \alpha(2)^{1/2}\,\Gamma(3/2) \end{aligned} \tag{25}$$

where Γ is the well-known Gamma function. Similarly, the relations between the sample L-moments and the three parameters of the Weibull model are derived as

$$\begin{aligned} t_3 &= \frac{l_3}{l_2} = \frac{\left(1 - 3\times 2^{-1/\kappa} + 2\times 3^{-1/\kappa}\right)}{\left(1 - 2^{-1/\kappa}\right)} \\ \lambda &= \frac{l_3}{\left(1 - 3\times 2^{-1/\kappa} + 2\times 3^{-1/\kappa}\right)\Gamma(1 + 1/\kappa)} \\ \rho &= l_1 - \lambda\,\Gamma(1 + 1/\kappa) \end{aligned} \tag{26}$$

The formulation for estimating the sample L-moments from a sample can be found in Hosking and Wallis (1997).

SAMPLE DATA

The data sets used in this study are obtained from two large-scale model tests performed on external turret-moored FPSOs. The first experiment represents a typical deepwater mooring system (water depth of more than 2000m) and the second experiment represents a typical mooring system designed for shallow-water conditions (water depth of less than 50m). A brief description of these mooring systems is provided below. Due to confidentiality considerations, more details of these projects cannot be reported.

- Deepwater Mooring System: the mooring system consists of 12 taut mooring lines grouped in three bundles of four mooring lines. The lines are designed in a chain-polyester-chain configuration.
- Shallow-water Mooring System: the mooring system consists of 12 catenary mooring lines grouped in four bundles of three mooring lines. The lines are designed in a chain-heavy chain-chain configuration.

The samples used here are measured during site-specific 100-year return period seastates with collinear wind, wave, and current. Each test was run for a duration equivalent to 9hr full-scale. The focus here is on the mooring line top tension and the vessel low-frequency offset. For this purpose, the tension measurements in the most loaded line (windward) and the least loaded line (leeward) as well as the vessel surge offset at the turret location are selected.
Fig. 1 shows the power spectrums of the top mooring line tension and the surge offset for the deepwater and shallow-water examples. Similarly, in Fig. 2, the power spectrums for the shallow-water example are presented. Note that in both figures, the spectrums are normalized by the sample variance.
As expected and also shown in both Fig.s 1~2, the mooring line low-frequency tension is responding to the slowly varying surge offset. The wave-frequency tension is mainly caused by the vertical motion at the location of the mooring line originated by vessel heave and pitch motions. The spectrum of the windward line tension of the deepwater mooring system shown in Fig. 1 indicates that the low-frequency tension has significantly larger contribution to the total tension than the wave-frequency tension. For the leeward line of the deepwater mooring system, the contribution of low-frequency and wave-frequency components is comparable. In the case of shallow-water mooring system, see Fig. (2), the total tension of both windward and leeward lines is dominated by the low-frequency tension component.

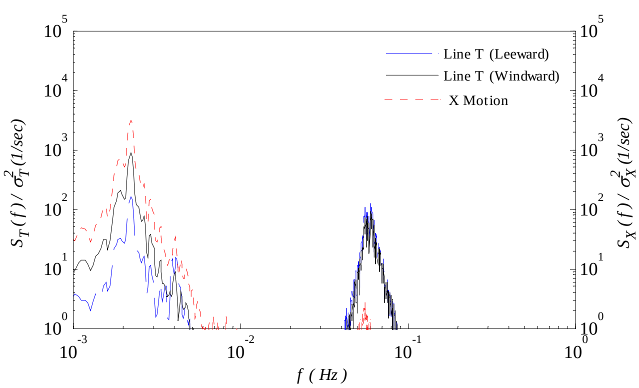


Fig. 1. The power spectrums of line tension and vessel motion of the deepwater example.

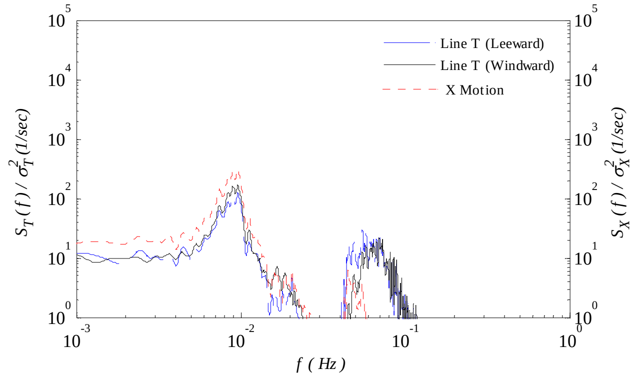


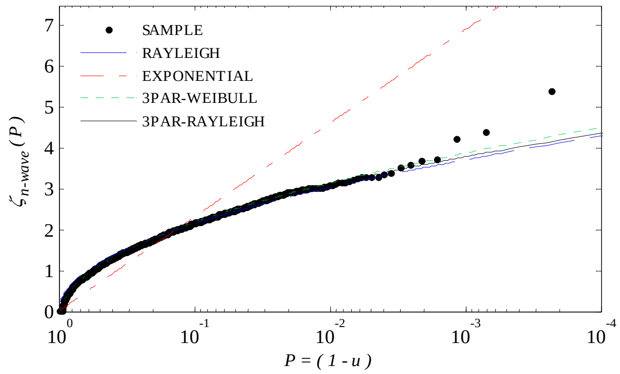
Fig. 2. The power spectrums of line tension and vessel motion of shallow-water example.

SAMPLE PROBABILITY DISTRIBUTIONS

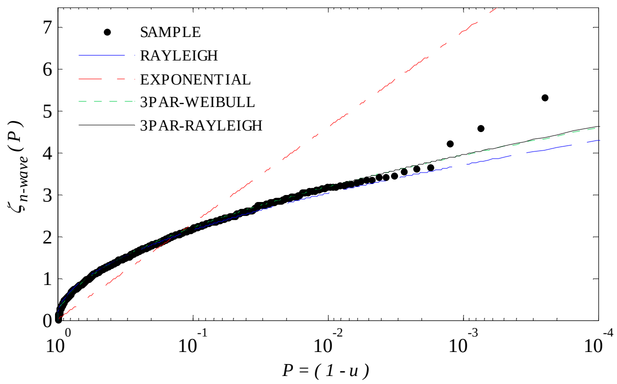
The characteristics of the quantile distribution of the samples introduced in the previous section are studied in this section. The quantile distribution (also known as inverse of CDF) defines the relation between the value of the random variable ζ_n and the probability of exceedance P defined as $1-u$ where $u(x)=F_{\zeta_n}(x)$.
In frequency domain analysis, the low-frequency and wave-frequency calculations are performed independently and then the extreme statistics of these two components are combined with the mean of the process to estimate the extreme statistics of the total process. The challenge in this approach is to model the correlation between the wave-frequency and low-frequency components correctly. Several studies have been done on methods of combining these components (see e.g. Naess 1989b and Liu and Bergdahl 1999). It is a common industry practice to either use the API formulation (API 2005) or conservatively estimate the extreme statistics of the total process from a simple summation of the extreme wave-frequency and extreme low-frequency components with the mean of the process. In time domain analysis, the statistics of the total process can be estimated directly from the sample results. Here, to better study the characteristics of the probability distributions, the probability distributions of wave-frequency tension, low-frequency tension, and the total tension are individually studied. In the case of vessel offset, only the probability distribution of the low-frequency surge offset is important.
Here, the normalized samples are obtained from

$$\zeta_{n-wave}=\frac{a_{n-wave}}{\sigma_{\eta-wave}},\quad \zeta_{n-low}=\frac{a_{n-low}-\mu_\eta}{\sigma_{\eta-low}},\quad \zeta_n=\frac{a_n-\mu_\eta}{\sigma_\eta} \tag{27}$$

where, η is the measured timeseries, μ_η is the mean of η, σ_η is the standard deviation of η, a_n are the amplitudes of η defined as the maximum observation between each two consecutive mean-crossings, and the terms “wave” and “low” refer to the estimates of the wave-frequency and low-frequency timeseries.
In Fig. 3 the quantile distribution of the normalized wave-frequency tension amplitudes of the deepwater mooring system are presented. Additionally, the quantile distributions of the Rayleigh model of linear amplitudes (Eq. 2), the Exponential model (Eq. 4), the three-parameter Weibull model (Eq. 7), and the three-parameter Rayleigh model (Eq. 10~12) are presented. Similarly, the quantile distributions of the normalized low-frequency tension amplitudes and normalized total tension amplitudes are presented in Fig.s 4~5, respectively. For the low-frequency tension, the quantile distribution of the Stansberg’s Exponential model (Eq. 6) is also presented.

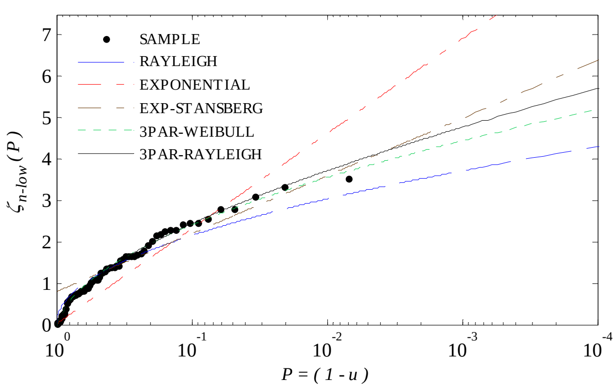


a. Windward line

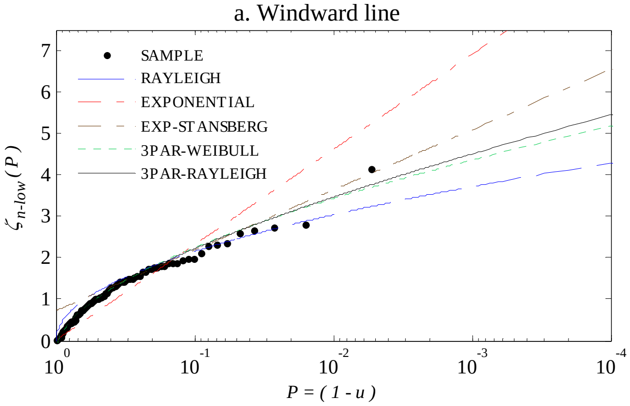


b. Leeward line

Fig. 3. Probability distribution of the amplitudes of wave-frequency tension of deepwater mooring system.

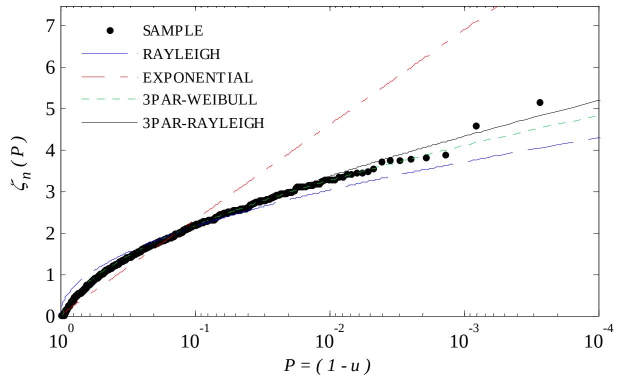


a. Windward line

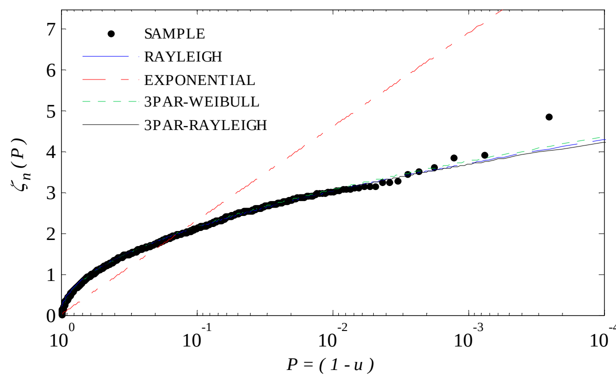


b. Leeward line

Fig. 4. Probability distribution of the amplitudes of low-frequency tension of deepwater mooring system.



a. Windward line



b. Leeward line

Fig. 5. Probability distribution of the amplitudes of total tension of deepwater mooring system.

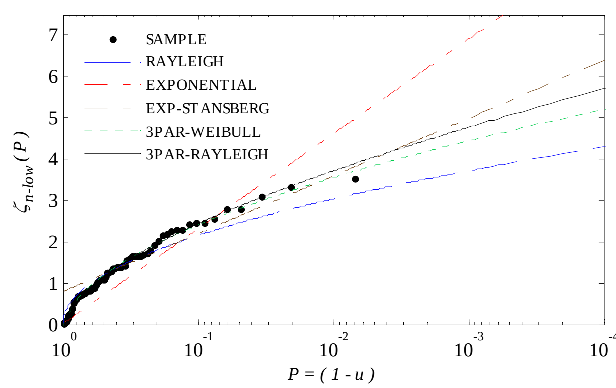


Fig. 6. Probability distribution of the amplitudes of slow drift motion of deepwater mooring system.

As shown in Fig. 3, except for the last three observations, the wave-frequency tension amplitudes of windward and leeward lines closely follow the Rayleigh distribution of linear amplitudes. For these samples, Exponential distribution significantly overestimates the amplitudes. It has been observed that the estimates of the three-parameter Weibull and the three-parameter Rayleigh model are in a close agreement and they are both successful in capturing the sample probability distribution.

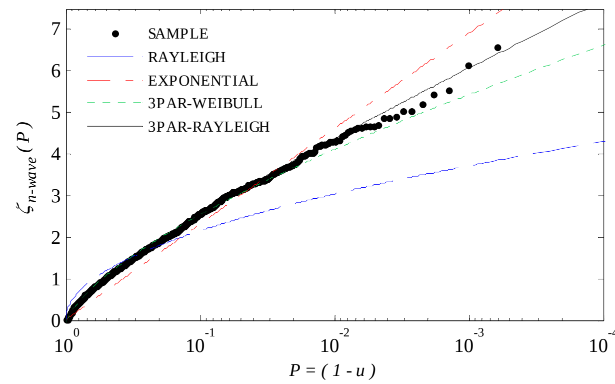
The low-frequency tension amplitudes of the windward line (Fig. 4) shows some level of non-linearity and the sample distribution deviates from the Rayleigh distribution. The low-frequency tension amplitudes of the leeward line, however, behave more linearly and follow the Rayleigh distribution more closely. For both windward and leeward examples, the Exponential distribution tends to overestimate the large amplitudes with small probability of exceedance. It has been observed that the Stansberg's Exponential model considerably performs better than the original Exponential model. As expected, the samples of low-frequency tension amplitudes contain limited number of observations which could cause some concerns about the performance of the three-parameter Weibull and the three-parameter Rayleigh distributions. However, as can be seen in Fig. 4 both models are successful in capturing the non-linearity in the low-frequency tension amplitudes of the windward line and capturing the distribution of the low-frequency tension amplitudes in leeward line.

The quantile distribution of the total tension of the windward line shows some deviation from the Rayleigh distribution which is an indication of weak non-linearity in this sample. The total tension of the leeward line can be very well approximated by the Rayleigh distribution model. For both total tension samples, Exponential model significantly overestimates the large amplitudes. The three-parameter Weibull and the three-parameter Rayleigh model found to be reasonably accurate in capturing the probability distribution of non-linear windward tension amplitudes and the linear leeward tension amplitudes.

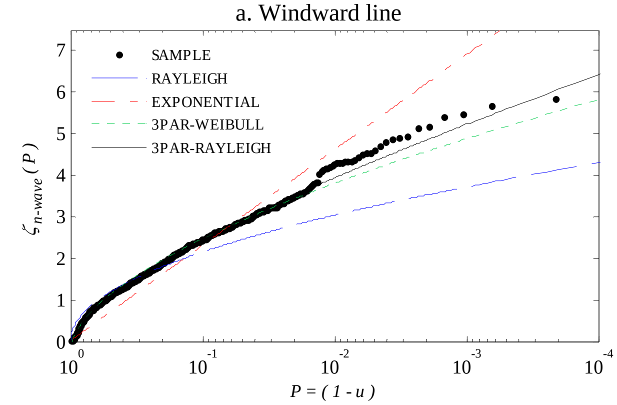
Similar to Fig. 4, the quantile distributions of the normalized low-frequency vessel surge amplitudes are shown in Fig. 6. As shown here, the quantile distribution of the normalized surge amplitudes is similar to that of the low-frequency tension amplitudes of the windward line.

The quantile distributions of the normalized wave-frequency tension amplitudes, low-frequency tension amplitudes, and total tension amplitudes of the shallow-water mooring system are presented in Fig.s 7 ~ 9. Comparing the distributions of tension amplitudes of the shallow-water mooring system to those of the deepwater mooring system, it can be concluded that the response of shallow-water mooring system is considerably more non-linear. The tension amplitudes, especially the low-frequency tension amplitudes, are almost exponentially distributed and the Rayleigh distribution significantly underestimates the amplitudes. The wave-frequency and low-frequency tension amplitudes and consequently the total tension amplitudes of the windward mooring line seems to be slightly more non-linear than the tension amplitudes measured in the leeward line. In this example, the Stansberg's Exponential model found to be a reasonable approximation of the tail distribution of the low-frequency tension amplitudes. In all studied cases, the three-parameter Weibull model and the three-parameter Rayleigh closely follow the sample distributions. As compared to the distributions estimated for the deepwater example, the difference between the tail of the three-parameter Weibull distribution and the tail of the three-parameter Rayleigh model is more sensible in the shallow-water examples. The three-parameter Rayleigh distribution consistently has heavier tail and predicts larger amplitudes with small probability of exceedance.

Similar to what was observed in the deepwater example, the quantile distribution of the normalized low-frequency surge amplitudes shown in Fig. 10 is similar to that of the low-frequency tension amplitudes of the windward line shown in Fig. 8.

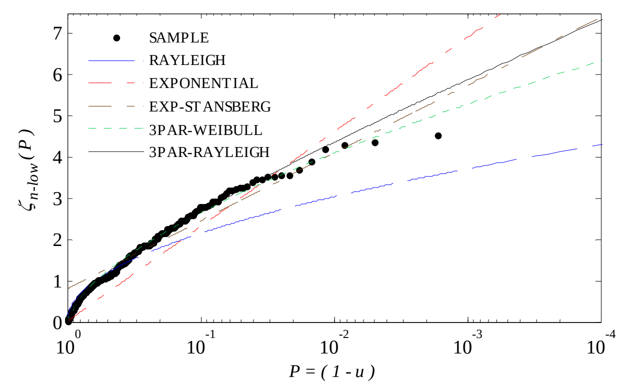


a. Windward line

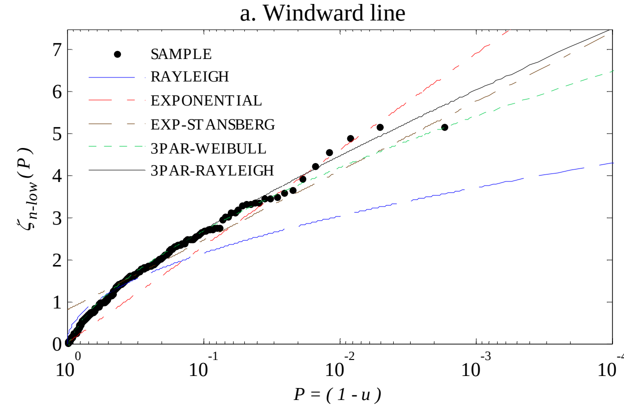


b. Leeward line

Fig. 7. Probability distribution of the amplitudes of wave-frequency tension of shallow-water mooring system.

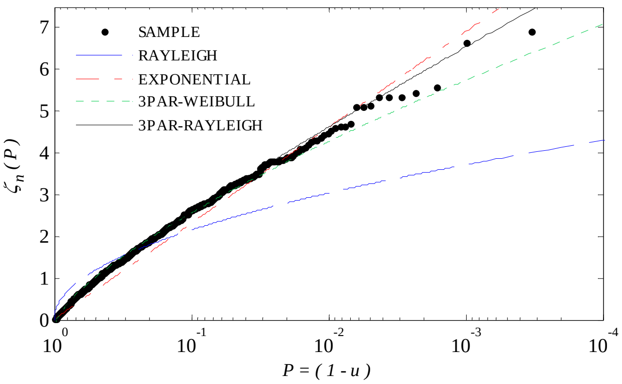


a. Windward line

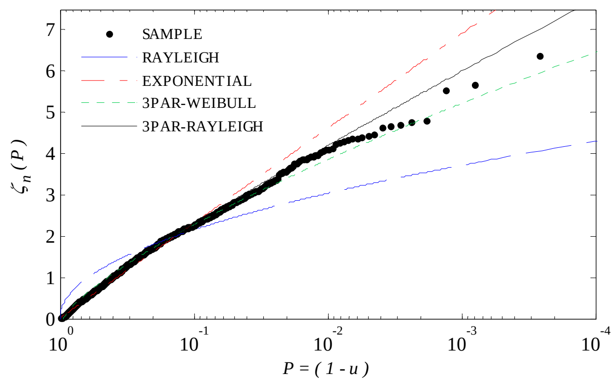


b. Leeward line

Fig. 8. Probability distribution of the amplitudes of low-frequency tension of shallow-water mooring system.



a. Windward line



b. Leeward line

Fig. 9. Probability distribution of the amplitudes of total tension of shallow-water mooring system.

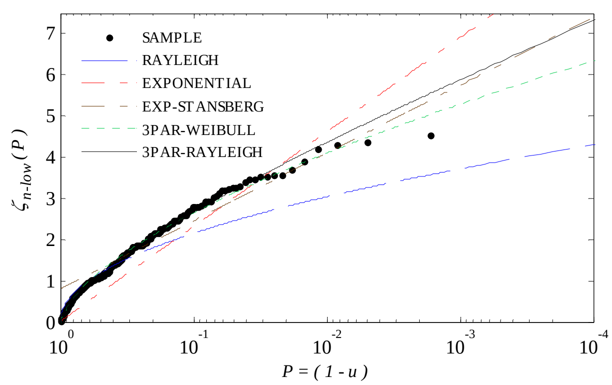


Fig. 10. Probability distribution of the amplitudes of slow drift motion of shallow-water mooring system.

The estimates of the expected maximum of the normalized mooring line tension in a 3hr storm for the deepwater and shallow-water mooring systems are provided respectively in Table 1~2. The estimates of the normalized expected maximum of the low-frequency surge offset are similar to those of the low-frequency tension of the windward line. In these tables, the sample estimate of the expected maximum is obtained by dividing the total 9hr timeseries into three 3hr samples and averaging the maximum of the three samples. The values shown in the parenthesis represents the range of the maximum amplitudes observed in the three 3hr samples. As shown here, the variability in the sample maximums is significant. In addition to the differences between the probability distribution of wave-frequency and low-frequency responses discussed before, the considerable difference in the number of cycles of these signals plays an important role in the difference between the normalized expected maximum of the two processes.

In Table 1~2, the predictions of different distribution models are compared. The results shown in these tables confirm the observations made earlier comparing the quantile distributions. In general, the extreme estimates of Rayleigh and Exponential distributions respectively define the lower and upper bounds of the estimates. Rayleigh distribution tends to underestimates the extreme statistics, which could result in significantly under predicted extremes in case of non-linear responses. The Exponential distribution tends to overestimate the extreme statistics, which could results in too conservative estimates for linear and weakly non-linear responses. It has been seen that the Stansberg's Exponential model is robust in predicting the extreme statistics of low-frequency responses and its estimates of extreme statistics match the sample estimates reasonably well. In the case of deep-water mooring system, the estimates of the three-parameter Weibull model and the three-parameter Rayleigh model are reasonably close and agree with the sample estimates. The predictions of the three-parameter Rayleigh distribution model for the highly non-linear amplitudes of the shallow-water mooring system are consistently larger than those of the three-parameter Weibull distribution. In the studied examples, the three-parameter Rayleigh distribution model seems to be performing better or as well as the three-parameter Weibull distribution.

Table 1. Normalized expected maximum of the mooring line tension of the deepwater mooring system.

Model	Windward			Leeward		
	Wave Freq.	Low Freq.	Total	Wave Freq.	Low Freq.	Total
Sample	4.2 (5.4 - 3.3)	3.2 (3.5 - 2.8)	4.2 (5.2 - 3.6)	4.1 (5.3 - 3.4)	3.1 (4.1 - 2.6)	3.9 (4.9 - 3.3)
Rayleigh	3.8	2.7	3.7	3.8	2.8	3.7
Exponential	7.1	3.8	7.0	7.1	4.0	7.1
3-Par. Rayleigh	3.8	3.3	4.3	4.0	3.1	3.7
3-Par. Weibull	3.9	3.1	4.1	4.0	3.1	3.8
Stansberg Exponential	--	3.4	--	--	3.5	--

Table 2. Normalized expected maximum of the mooring line tension of the shallow-water mooring system.

Model	Windward			Leeward		
	Wave Freq.	Low Freq.	Total	Wave Freq.	Low Freq.	Total
Sample	5.8 (7.3 - 4.9)	4.1 (4.5 - 3.6)	5.8 (6.9 - 5.1)	5.6 (5.8 - 5.4)	5.0 (5.2 - 4.6)	5.4 (6.4 - 4.4)
Rayleigh	3.8	3.2	3.7	3.8	3.2	3.7
Exponential	7.3	5.2	6.8	7.2	5.2	7.0
3-Par. Rayleigh	6.3	4.7	6.4	5.4	4.8	6.1
3-Par. Weibull	5.6	4.4	5.7	5.0	4.5	5.3
Stansberg Exponential	--	4.5	--	--	4.5	--

CONCLUSIONS

The main goal of this study was to study the characteristics of the probability distribution of mooring line tension and low-frequency vessel horizontal offset of turret moored FPSO. For this purpose, the sample data sets obtained from two experimental model tests of external turret moored systems in extreme environmental conditions are used. The examples studied here represent typical deepwater and shallow-water mooring systems. In the case of mooring line tension, the behavior of the wave-frequency, low-frequency, and total tension is individually studied. The statistics of both most loaded line (windward) and least loaded line (leeward) are presented. It has been observed that the response of the shallow-water mooring system is considerably more non-linear than the response of deepwater mooring system. In both mooring systems, the non-linearity is more sensible in the windward line than the leeward line. In the case of deepwater mooring system the non-linearity in the mooring line tension is mainly sourced from the low-frequency tension, while in the shallow-water example both wave-frequency and low-frequency tension components are highly non-linear. In both shallow-water and deepwater designs the characteristics of the distribution of low-frequency surge motion is similar to those of low-frequency tension of the windward line.

In order to estimate the probability distribution of non-linear random variables four distribution models, i.e. Exponential, Stansberg's Exponential, three-parameter Weibull, and three-parameter Rayleigh, are utilized. The models are used to estimate the sample distribution and the statistics are compared to those of Rayleigh distribution of linear amplitudes. The Rayleigh distribution is commonly used in offshore industry to estimate the extreme statistics. Using the measured sample data, it is confirmed that the Rayleigh distribution significantly underestimates the extreme statistics of non-linear random variable. The Exponential probability distribution defined the upper limit for the studied examples and overestimated the extreme statistics when the response is not highly non-linear. Stansberg's modification to Exponential distribution found to improve the performance of the original model. The three-parameter Rayleigh distribution model was consistently successful in estimating the probability distribution of data. The model has the flexibility to capture the distribution of linear and non-linear variables. In most studied cases, the estimates of the three-parameter Rayleigh distribution models were reasonably close to those of the widely used three-parameter Weibull distribution. The difference between the tails of the two models was sensible for highly non-linear responses of the shallow-water system. For those samples, the three-parameter Rayleigh distribution model estimated larger expected maximum, which were closer to the sample estimates.

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