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APPLICATION OF SEMI-EMPIRICAL PROBABILITY DISTRIBUTIONS IN WAVE-STRUCTURE INTERACTION PROBLEMS

Amir H. Izadparast
Research Engineer, SOFEC Inc.
Houston, Texas, USA

John M. Niedzwecki
Professor, Zachry Department of Civil Engineering, Texas A&M University.
College Station, Texas, USA

ABSTRACT

Ocean engineers are routinely faced with design problems for coastal and deepwater structures that must survive a wide range of environmental conditions. One of the most challenging problems in the field of ocean engineering is the accurate characterization and modeling of the interaction of ocean waves with these offshore structures. The random characteristic of ocean environment requires engineers to consider the effects of random variability of the pertinent variables in their predictive models and design processes. Thus, for ocean engineering purposes, one needs to have accurate estimates of the probability distribution of the key random variables that will be used in sensitivity studies, reliability analysis, and risk assessment in the design process.

In this study, a family of semi-empirical probability distribution is developed based on the quadratic transformation of linear random variable assuming that the linear random variable follows a Rayleigh distribution law. The estimates of model parameters are obtained from two moment based parameter estimation methods, i.e. method of moments and method of linear moments. The studied semi-empirical distribution can be applied to estimate the probability distribution of a wide range of non-linear random variables in the fields of ocean wave mechanics and wave-structure interaction. As examples, the application of the semi-empirical model in estimation of probability distribution of: a) ocean wave power, b) ocean wave crests interacting with an offshore structure is illustrated. For this purpose, numerically generated timeseries and experimentally measured data sets are utilized.

INTRODUCTION

The wave-structure interaction problem is a major design challenge for ocean engineers in both benign and extreme environmental conditions. The interaction between highly energetic waves and the structure may result in unanticipated behavior including large structural motions, wave impact loading, and deck inundation. At some offshore sites, mildly energetic ocean waves may be converted to usable energy, utilizing Wave Energy Convertor (WEC) devices that capture the kinematic energy of the wave motions. Due to the random characteristic of ocean environment, ocean engineers are required to consider the effects of random variability of the pertinent variables in their predictive models and design processes. For the thorough study of wave-structure interaction phenomena, one requires robust methods to estimate the probability distribution of complex non-linear random variables.

In the theory of statistics, parametric distribution models are considered as a major family of probability distributions in which a distribution function consists of an underlying structural form that is dependent upon finite number of parameters. Two main approaches are commonly utilized in different engineering fields to specify the structural form and estimate the parameters of a parametric model, i.e. theoretical approach and empirical approach. In the theoretical approach, the structural form and the model parameters are derived based on a mathematical model that approximates the physics governing the process. For example, Tayfun [1-3] utilized a special form of second-order Stokes' wave theory to derive the probability distribution of weakly non-linear ocean wave crests. In the empirical approach, it is assumed that the random variable follows a standard probability distribution e.g. Gaussian, Weibull, Rayleigh, etc. and the unknown parameters are estimated empirically using sample data. For example, Forristall [4, 5] utilized numerically generated and measured wave data to show that the probability distribution of second-order ocean wave crests can be reasonably presented by Weibull distribution. Stansell [6, 7] showed that the extreme wave crests measured in the North Sea follow the generalized-Pareto distribution. The main advantages of the theoretical approach are: (a) the probability distribution and the model parameters reflect some physical insight, and (b) the parameter estimation requires limited information about the process. However, the theoretical models lack flexibility and their efficiency decreases when the assumptions are not fully met. The empirical models have found to be more flexible in capturing the probability distribution of data and have a wider range of applicability. However, the empirical model and the parameters do not have clear connection to the physics of the process. The application of empirical models has become an important option with availability of high quality data sets from full-scale measurements, experimental model tests, and calibrated numerical models.

In recent studies, significant attention is given to the semi-empirical probability distribution functions that incorporate empirically derived parameters to theoretically derived distribution forms [8, 9]. What separates the semi-empirical models from routinely used empirical distributions is that the structural form of the semi-empirical model is developed from a mathematical approximation of the random process and therefore the underlying structural form and the associated model parameters provide some physical insight. Moreover, the empirically estimated parameters improve the flexibility of the model in capturing the nature of data and therefore the semi-empirical model performance is comparable to that of empirical model.

The three-parameter distribution model derived in this study is based upon the quadratic transformation of linear random variables first introduced by Tayfun [1] to study the behavior of weakly non-linear wave crests. Tayfun [1] theoretically developed a Rayleigh-Stokes probability distribution assuming that the linear process to be narrow-banded and Rayleigh distributed. Additionally, it was assumed that the non-linear process could be approximated by second-order Stokes' expansion, the first-order and the second-order terms are phase-locked, and that the various terms are in phase. This theoretical distribution model was a one-parameter probability distribution and the model parameter was estimated from its theoretical relation with the significant wave height and mean period. Tyfun's model was subsequently modified by other researchers [3, 10-11], and their research findings indicated a reasonable agreement between the Rayleigh-Stokes one-parameter model statistics and the sample statistics of simulated and measured ocean wave crests. Following a similar approach, Kriebel [12] developed a two-parameter Rayleigh-Stokes model to study the probability distribution of wave run-up over large bottom-mounted vertical columns in which he incorporated the amplification factor obtained from linear diffraction theory. Although the theoretical Rayleigh-Stokes models are important developments in the field of ocean engineering, they have limited flexibility in capturing the probability distribution of complex random variables.

In this study, the semi-empirical approach is utilized to develop a three-parameter Rayleigh-Stokes model in which the structural form of the distribution is derived theoretically while the estimates of the model parameters are obtained from two moment based parameter estimation methods, i.e. method of moments and method of linear moments. The semi-empirical Rayleigh-Stokes model can be applied to estimate the probability distribution of a wide range of non-linear random variables in the fields of ocean wave mechanics and wave-structure interaction.

MODEL DEVELOPMENT

Based on the second-order Stokes wave theory, the crests (or troughs) ζ_n of a weakly non-linear and narrow banded process η_n can be approximated from [1]

$$\zeta_n = \alpha\, \zeta + \beta\, \zeta^2 + \gamma \tag{1}$$

where, ζ is the crests (or troughs) of the narrow-banded linear random variable η, α is the amplification of the linear term, β is the amplification of the quadratic term, and γ is the remaining shifting between linear and non-linear variables. Here, crests and troughs are defined as maximum and minimum elevations between each two consecutive zero-upcrossings, respectively (see Fig. (1)). In this model β and γ have real values, α is a positive real value, and in order to satisfy the weakly non-linear assumption $|\beta| << \alpha$.

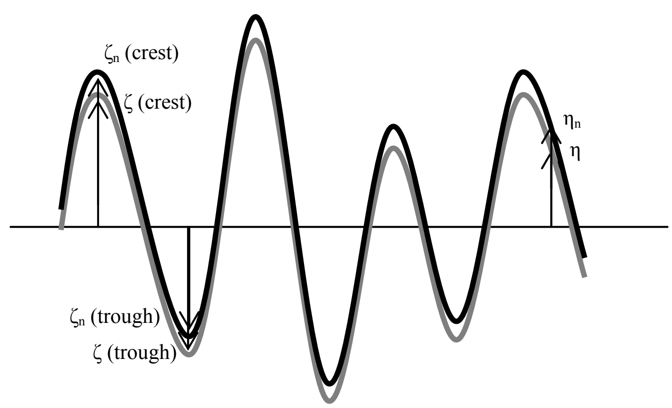

Figure 1-Variable definition.

It can be shown that the linear term ζ follows a Rayleigh distribution law [13] with probability density function (PDF) of

$$f_\zeta(x) = x \exp\left(-x^2/2\right) \tag{2}$$

From that and application of the random variable transformation rule, the PDF, the cumulative distribution function (CDF), and the quantile distribution of the non-linear variable ζ_n for $\beta > 0$ is obtained as

$$f_{\zeta_n}(x) = \frac{\chi-\alpha}{2\beta\chi}\exp\left(-\frac{(\chi-\alpha)^2}{8\beta^2}\right)$$

$$F_{\zeta_n}(x) = 1-\exp\left(-\frac{(\chi-\alpha)^2}{8\beta^2}\right) \quad (3)$$

$$x_{\zeta_n}(u) = \gamma - 2\beta\ln(1-u) + \alpha\,(-2\ln(1-u))^{1/2}$$

where, u is the probability of exeedance $0 \le u < 1$, χ is defined as,

$$\chi = \left(\alpha^2 + 4\beta(x-\gamma)\right)^{1/2} \quad (4)$$

and $x > \gamma$. In the case of $\beta < 0$, the probability distribution functions are derived in the form of

$$f_{\zeta_n}(x) = \frac{1}{2\beta\chi}\left[(\chi-\alpha)\exp\left(-\frac{(\chi-\alpha)^2}{8\beta^2}\right)H_\gamma(x) + -(\chi+\alpha)\exp\left(-\frac{(\chi+\alpha)^2}{8\beta^2}\right)\right] \quad (5)$$

$$F_{\zeta_n}(x) = \left[1-\exp\left(-\frac{(\chi-\alpha)^2}{8\beta^2}\right)\right]H_\gamma(x) + \exp\left(-\frac{(\chi+\alpha)^2}{8\beta^2}\right)$$

where $H_\gamma(x)$ is the step function and has a value of unity for $x \ge \gamma$ and is zero for $x < \gamma$. The distributions in Eq. (5) are defined for $-\infty < x < \gamma - \frac{\alpha^2}{4\beta}$ while only $\gamma < x < \gamma - \frac{\alpha^2}{4\beta}$ is physically meaningful. The exact analytical form of the quantile function is not available for the Rayleigh-Stokes model with $\beta < 0$. An approximate estimate for the quantile function can be obtained assuming that (α-χ) << (χ+α) and consequently the third term in the $F_{\zeta_n}(x)$ can be ignored with respect to the second term; specifically,

$$x_{\zeta_n}(u) \approx \gamma - 2\beta\,\ln(1-u) + \alpha\,(-2\ln(1-u))^{1/2} \quad (6)$$

In the special case where α = 0, the Rayleigh-Stokes model simplifies to an exponential distribution with distribution functions,

$$f_{\zeta_n}(x) = \frac{1}{2\beta}\exp\left(-\left(\frac{x-\gamma}{2\beta}\right)\right), \qquad x > \gamma$$

$$F_{\zeta_n}(x) = 1-\exp\left(-\left(\frac{x-\gamma}{2\beta}\right)\right), \quad (7)$$

$$x_{\zeta_n}(u) = \gamma - 2\beta\,\ln(1-u).$$

The quantile distribution of Rayleigh-Stokes model for $1.0 \le \alpha \le 1.6$, $-\alpha/10 \le \beta \le \alpha/10$, and $\gamma = 0$ is shown in Fig. (2). Note that positive and negative γ values shift the entire quantile distribution up and down, respectively.

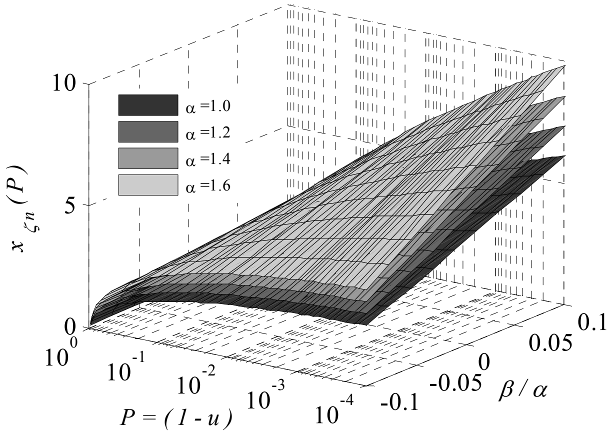

Figure 2-Rayleigh-Stokes quantile distribution.

PARAMETER ESTIMATION

Here, two moment based estimation methods, i.e. method of moments (MoM) and method of linear moments (MoLM) are utilized to obtain the estimates of the unknown Rayleigh-Stokes model parameters α, β, and γ. In both methods, a system of three equations is obtained by equating the distribution statistics with their corresponding sample statistics. The system of equations is then solved for the model parameters. These methods are briefly discussed in the followings.

Method of Moments

For a random variable X, the first distribution moment, i.e. mean, is defined as [14]

$$\mu_1(X) = E(X) \quad (8)$$

and the nth moment is defined as

$$\mu_n(X) = E(X-\mu)^n \quad (9)$$

where, $E(g(X))$ is the expectation of the function $g(X)$ and is obtained from

$$E(g(X)) = \int_0^1 g(x(u))\,du \quad (10)$$

From that, the distribution moments can be defined in the form of,

$$\mu_1(X) = \int_0^1 x(u)\,du \qquad n=1$$

$$\mu_n(X) = \int_0^1 (x(u)-\mu_1)^n\,du \qquad n>1 \quad (11)$$

The mean $\mu_1(X)$ represents the centroid of the distribution and the variance $\sigma_X^{\,2} = \mu_2(X)$ is a measure of the distribution dispersion around its center. Other useful moments are the dimensionless third and fourth moments, respectively called skewness $s_X = \mu_3(X)/\sigma_X^{\,3}$ and coefficient of excess kurtosis $K_X = \mu_4(X)/\sigma_X^{\,4} - 3$. Utilizing the quantile function of the Rayleigh-Stokes model given in Eq.s (3) and (6) in the integrals in Eq. (11), the relation between the first three distribution moments and the model parameters are derived in the form of

$$\mu_1(\zeta_n) = \gamma + 2\beta + \alpha\sqrt{2}\Gamma(3/2)$$
$$\mu_2(\zeta_n) = 4\beta^2 + \alpha\beta(2)^{3/2}\Gamma(3/2) + 2\alpha^2(1-\pi/4)$$
$$\mu_3(\zeta_n) = 16\beta^3 + 9\sqrt{2}\alpha\beta^2\,\Gamma(3/2) + 12\alpha^2\beta\left(1-\Gamma^2(3/2)\right) + \alpha^3(2)^{3/2}\left(2\Gamma^3(3/2) - \frac{3}{2}\Gamma(3/2)\right) \quad (12)$$

where Γ is the well-known Gamma function. As shown in this equation, the nth moment is a polynomial of degree n of the parameters α and β, and as expected γ is only shifting the distribution mean. It should be noted that the relations in Eq. (12) can be used for the special case with $\alpha = 0$ (Eq. (7)).

For a data set $x_1, x_2, \ldots, x_{N_s}$ of size N_s, the unbiased estimates of the first three sample moments are obtained respectively from [14]

$$\hat{\mu}_1(X) = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i$$
$$\hat{\mu}_2(X) = \hat{\sigma}_X{}^2 = \frac{1}{(N_s-1)}\sum_{i=1}^{N_s}(x_i - \hat{\mu}_1)^2 \quad (13)$$
$$\hat{\mu}_3(X) = \frac{N_s}{(N_s-1)(N_s-2)}\sum_{i=1}^{N_s}(x_i - \hat{\mu}_1)^3$$

In order to estimate the three model parameters of the Rayleigh-Stokes distribution α, β, and γ with MoM, the first three distribution moments (Eq. (12)) are equated with their unbiased sample estimators (Eq. (13)), which gives a system of equations to be solved for the unknown parameters. An iterative numerical solver is applied to solve the system of equations. For the exponential distribution defined in Eq. (7), the model parameters can be estimated directly from,

$$\alpha = 0,$$
$$\hat{\beta} = \frac{\left(\hat{\mu}_2(\zeta_n)\right)^{1/2}}{2}, \quad (14)$$
$$\hat{\gamma} = \hat{\mu}_1(\zeta_n) - 2\hat{\beta}$$

Method of L-Moments

Linear moments (L-moments) are developed from modifying the probably-weighted moments (PWM) formerly introduced by Greenwood et al. [15]. The main difference between ordinary moments and PWMs is that ordinary moments give greater weight to the extreme tails of the distribution. Therefore, distribution moments are more successful in representing the extreme values. However, the sample moments are highly affected by unexpectedly large observations and consequently high order moments are considerably more biased than the corresponding probability-weighted moments. PWMs are considered the desirable sample estimators for extreme analysis when the sample sizes are limited.

PWMs, as alternatives to ordinary moments, have been used in the field of probability distribution parameter estimation [e.g. 16-17]. However, it is difficult to directly connect PWMs to the characteristics of the probability distribution function, e.g. shape and scale. Hosking introduced L-moments from a linear combination of PWMs to overcome this issue. For a random variable X with quantile function of $x(u)$, the distribution L-moments are obtained from integration of quantile function multiplied by an orthogonal function [14], specifically

$$\lambda_n(X) = \int_0^1 x(u)\,P^*_{n-1}(u)\,du \quad (15)$$

where the orthogonal function is the shifted Legendre polynomials $P^*{}_n(u)$

$$P^*_n(u) = \sum_{k=0}^{r} p^*_{n,\,k}\,u^k \quad (16)$$

and the function coefficients are obtained from

$$p^*_{n,k} = \frac{(-1)^{n-k}(n+k)!}{(k!)^2(n-k)!} \quad (17)$$

By definition, λ_1 is the L-location or mean of the distribution, λ_2, $\tau_3 = \lambda_3/\lambda_2$ are L-scale, and L-skewness respectively, and are analogous to the ordinary standard deviation and skewness. A more complete definition of the linear moments and their characteristics can be found in (14).

Applying the quantile function of Rayleigh-Stokes model, Eq.s (3) and (6), in definition of linear moments (Eq. (15)), the relations between the distribution moments and the model parameters are derived as

$$\lambda_1(\zeta_n) = \gamma + 2\beta + \alpha(2)^{1/2}\Gamma(3/2)$$
$$\lambda_2(\zeta_n) = \beta + \alpha\left(2^{1/2}-1\right)\Gamma(3/2) \quad (18)$$
$$\lambda_3(\zeta_n) = \beta/3 + \alpha\left(2^{1/2} - 3 + (8/3)^{1/2}\right)\Gamma(3/2)$$

As shown here, L-moments are linear functions of parameters α and β and the shifting parameter γ only appeared in the L-location.

The sample L-moment l_n of an ordered sample $x_{1:N_s} \le x_{2:N_s} \le \ldots \le x_{N_s:N_s}$ of size N_s is defined as [14]

$$l_{n+1}(X) = \sum_{k=0}^{n} p^*_{n,\,k}\,\varsigma_n \qquad n = 0, 1, \ldots, N_s - 1 \quad (19)$$

where

$$\varsigma_n = N_s{}^{-1}\binom{N_s-1}{n}^{-1}\sum_{j=n+1}^{N_s}\binom{j-1}{n}x_{j:N_s} \quad (20)$$

and the brackets denote binomial coefficients. Note that, the sample L-moment l_n is an unbiased estimator of the distribution L-moments λ_n. Equating the first three linear moments with their corresponding sample statistics, the estimates of Rayleigh-Stokes model parameters are obtained in the form of

$$\hat{\alpha} = 4.1394\left(l_2\left(\zeta_n\right) - 3 l_3\left(\zeta_n\right)\right)$$
$$\hat{\beta} = \left(l_2\left(\zeta_n\right) - \left(2^{1/2} - 1\right)\hat{\alpha}\,\Gamma\left(3/2\right)\right) \quad (21)$$
$$\hat{\gamma} = l_1\left(\zeta_n\right) - 2\hat{\beta} - \hat{\alpha}\left(2\right)^{1/2}\Gamma\left(3/2\right)$$

For the special case with $\alpha = 0$ (Eq. (7)), the relations simplify into,

$$\alpha = 0,$$
$$\hat{\beta} = l_2\left(\zeta_n\right)/R,$$
$$\hat{\gamma} = l_1\left(\zeta_n\right) - 2\hat{\beta}R \quad (22)$$

Izadparast and Niedzwecki [18] compared the performance of MoM and MoLM for the three-parameter Rayleigh-Stokes model utilizing Monte-Carlo simulations. It was concluded that both methods perform reasonably well for large samples ($N_s \geq 1000$) while MoLM is more suited for samples with limited size. It was observed that for both methods, large enough samples ($N_s \geq 300$) are required to obtain reasonably accurate estimates of the non-linear term β.

Extreme statistics

Assuming that ζ_n are independent identically distributed (i.i.d) random variables, the PDF and CDF of the maxima $\zeta_{\max}$ in N events can be obtained from the ordered value statistics theory, specifically, (19)

$$f_{\zeta_{\max}}\left(x\right) = N f_{\zeta_n}\left(x\right)\left[F_{\zeta_n}\left(x\right)\right]^{N-1}$$
$$F_{\zeta_{\max}}\left(x\right) = \left[F_{\zeta_n}\left(x\right)\right]^{N} \quad (23)$$

For large number of N, it can be shown that the Rayleigh-Stokes probability distribution function belongs to the Gumbel maximal domain of attraction and the asymptotic form of $x_{\zeta_{\max}}\left(u\right)$ can be represented by

$$x_{\zeta_{\max}}\left(u\right) = a_N - b_N \ln\left(-\ln\left(u\right)\right) \quad (24)$$

where a_N and b_N are the extreme distribution parameters and can be estimated from their relation with Rayleigh-Stokes parameters

$$a_N = \gamma + 2\beta \ln\left(N\right) + \alpha\left(2\ln\left(N\right)\right)^{1/2}$$
$$b_N = 2\beta + \alpha\left(2\ln\left(N\right)\right)^{-1/2} \quad (25)$$

Assuming that $\zeta_{\max}$ follows the Gumbel probability distribution function, the first three moments of $\zeta_{\max}$ are,

$$\mu_1\left(\zeta_{\max}\right) = E\left(\zeta_{\max}\right) = a_N + b_N \gamma_{EM}$$
$$\mu_2\left(\zeta_{\max}\right) = \frac{\pi^2}{6} b_N^2 \quad (26)$$
$$\mu_3\left(\zeta_{\max}\right) = 2\zeta_R\left(3\right) b_N^3$$

where $\gamma_{EM} \approx 0.5772$ is the Euler-Mascheroni constant and $\zeta_R\left(z\right)$ is the Riemann zeta function that is $\zeta_R\left(3\right) \approx 1.2021$ at $z = 3$. Similarly, the first three L-moments of $\zeta_{\max}$ are derived

$$\lambda_1\left(\zeta_{\max}\right) = a_N + b_N \gamma_{EM}$$
$$\lambda_2\left(\zeta_{\max}\right) = b_N \ln\left(2\right) \quad (27)$$
$$\lambda_3\left(\zeta_{\max}\right) = b_N\left(2\ln\left(3\right) - 3\ln\left(2\right)\right)$$

In Fig. 3, the quantile distribution of Rayleigh-Stokes maxima in $N = 1000$ incidents is presented. The distributions in Fig. 3 are estimated from applying the Rayleigh-Stokes distribution in Eq. (23). It should be noted that for $\beta < 0$ the Rayleigh-Stokes probability distribution function has an upper bound which is considered in the calculation of distributions shown in Fig. 3.

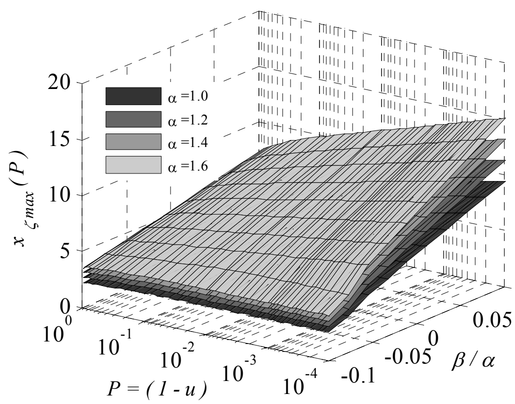

Figure 3-quantile distribution of maxima in 1000 events of the Rayleigh-Stokes model.

CASE STUDIES

Two examples are studied here to illustrate the application of Rayleigh-Stokes model for estimating the probability distribution of non-linear random variables. In the first example, the semi-empirical Rayleigh stokes model is applied to estimate the probability distribution of measured wave crests in the vicinity of an offshore platform. In the second example, the probability distribution of random wave power is estimated.

Disturbed wave crests

The theoretical one-parameter Rayleigh-Stokes model has been widely used to estimate the probability distribution of undisturbed ocean wave crests. As incident waves get closer to a floating structure they interact with the diffracted and radiated waves from the structure. As a result, the disturbed waves in the area close to a structure become more non-linear and more complex than the original incident waves. It has been observed that the one-parameter Rayleigh-Stokes model is not capable of capturing the complex nature of the disturbed wave crests [8]. However, the theoretical form of the Rayleigh-Stokes model is still valid for the disturbed wave crests. Thus, the semi-empirical Rayleigh-Stokes distribution seems to be an appropriate model for estimating the probability distribution of disturbed wave crests.

Wave power

According to the linear wave theory, the wave power per unit crest length P of regular seas can be expressed as the product of the wave group velocity C_g and the total average wave energy per unit surface area E, see e.g. [20]. For the deep and shallow water limits, indicated with the subscript d and s, respectively, the wave powers can be expressed as,

$$P_d = \frac{\rho g^2}{32\pi}\, T H^2 \qquad d/\lambda_\eta > 1/2$$
$$P_s = \frac{\rho g^{3/2}}{8} d^{1/2} H^2 \qquad d/\lambda_\eta < 1/20 \tag{28}$$

where ρ is the water density, g is the gravitational acceleration (9.81m/sec), T is the wave period, H is the wave height, d is the water depth, λ is the wave number $\lambda_\eta = 2\pi/L$, and L is the wave length. The dimensionless form of wave power for deep and shallow water limits is defined as (21),

$$\mathrm{p}_{(d\,or\,s)} = P_{(d\,or\,s)} \big/ \hat{P}_{(d\,or\,s)} = \beta\zeta^2 \tag{29}$$

where $\zeta = H / \sigma_\eta$, $\sigma_\eta = H_s /4$ is the standard deviation of linear and narrow-banded surface wave elevation,

$$\hat{P}_d = \frac{\rho g^2}{64\pi}\, T_p H_s^2 \tag{30}$$

and

$$\hat{P}_s = \frac{\rho g^{3/2}}{16}\, d^{1/2} H_s^2 \tag{31}$$

Parameter β is a function of wave period in deepwater limit and has a constant value of 1/8 for shallow water limit. It can be shown that β asymptotes to 1/8 for narrow banded waves in deepwater as well.

Izadparast, Niedzwecki [21] derived the theoretical probability distribution of wave power for different limits. For a simplified case of narrow-banded waves, Eq. (29) can be considered as a special case of Eq. (1) with $\alpha = 0$. As an alternative to the theoretical model, Rayleigh-Stokes model can be used as a data analysis tool to estimate the variability in the wave power data.

DATA ANALYSIS AND RESULTS

Disturbed wave crests

To evaluate the performance of the Rayleigh-Stokes model in capturing the probability distribution of disturbed wave crests, the data sets obtained from a mini-TLP model test investigating the behavior of the structure in an extreme environment are utilized. The model tests were performed in the wave basin at Offshore Technology Research Center (OTRC). Details of this experiment can be found in the articles by Niedzwecki et al. [22] and Teigen, Niedzwecki, and Winterstein [23]. In this study only the measurements of a 3-hr seastate generated from JONSWAP spectrum with significant wave height $H_s = 4.0m$, peak period of $T_p = 16s$ and peakedness factor $\gamma_s = 2.0$ are analyzed. The seastate represents the 100-year design wave conditions off the West Africa coastline for which the mini-TLP was originally designed. The data set used here is measured in the long crested waves attacking the structures in the heading seas direction (see Fig. 4). The wave timeseries measured over the first pontoon (at A2 in Fig. 4) are used in the zero-crossing analysis to obtain the wave crests sample. The wave crest sample is normalized by the first order standard deviation of incident waves σ_η that is estimated from its relation with the significant wave height $\sigma_\eta = H_s/4 = 1.0$. Next, the sample distribution is estimated by using the Kernel probability estimation method [24] while the sample tail distribution is modified with the generalized-Pareto distribution. The sample distribution is then utilized to generate 20,000 bootstrap samples to obtain the 95 percent confidence limits (CL) and estimate the root-mean-squared error (RMSE). More details of the non-parametric probability distribution estimation and bootstrap analysis can be found in [25-27].

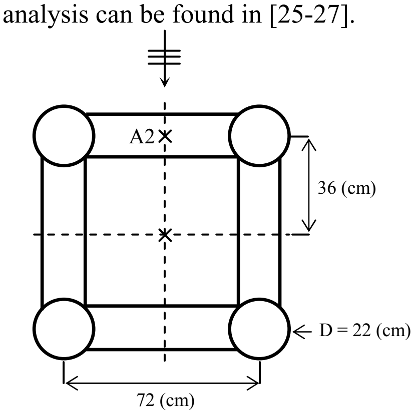

Figure 4- schematic view of mini-TLP model test (dimension scale 1/40).

In Fig. 5 (a), the sample quantile distribution is compared with those of the Rayleigh-Stokes distribution with the parameters estimated from MoM and MoLM. As shown in Fig. 5, the Rayleigh-Stokes model is successful in capturing the probability distribution of data and the estimates of MoM and MoLM converge considerably well. Fig. 5 (b) shows the RMSE distribution of the Rayleigh-Stokes quantile as a function of the probability of exceedance. As expected the RMSE increases on the distribution tail. However, as shown in Fig. 5 (b) for this example, the RMSE of Rayleigh-Stokes estimates of a relatively large wave crest (~ H_s = 4.0m) is considerably small (<0.15m). This is an indication that the semi-empirical Rayleigh-Stokes model can be used to model the probability distribution of rare events on the sample tail. Utilizing the estimates of the Rayleigh-Stokes model parameters in Eq. (25) and (26), the estimates of expected maximum wave crests are calculated as 4.72 and 4.76 (equivalent to 4.72m and 4.76m) respectively for MoM and MoLM. As expected, due to the relatively large sample size in the studied case (N_s = 810), the estimates of MoM and MoLM are almost identical.

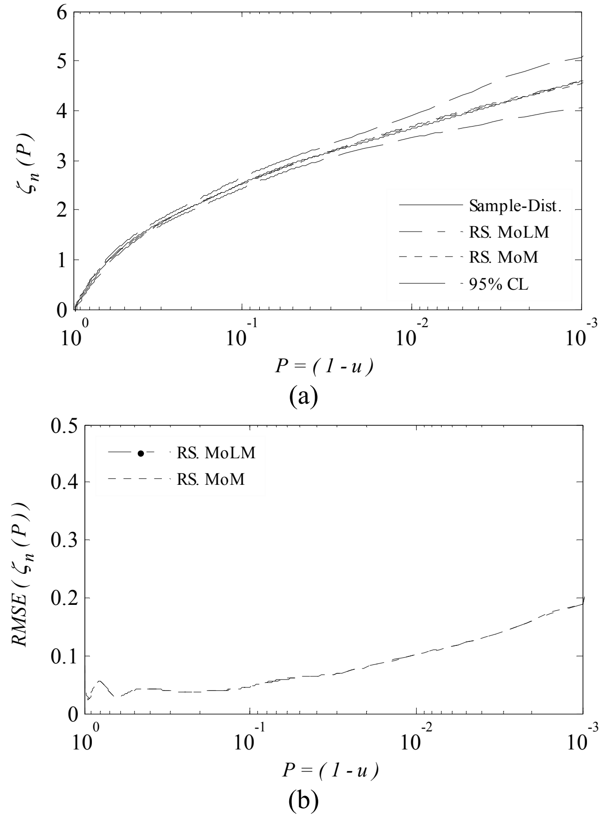

Figure 5- (a) Quantile distributions of wave crests at A2, (b) RMSE distribution of quantile estimates.

Wave power

The sample data set of wave power timeseries can be obtained from time domain analysis of simulated or measured wave timeseries. Basically, the observations are developed from a zero-crossing analysis which in turn allows a wave-by-wave estimate of the average wave power in each wave cycle. More specifically, the wave periods T_i and associated wave heights H_i are obtained from the zero-crossing analysis of the surface elevation time series. Next, for each pair of H_i and T_i the wave power P_i is estimated using the appropriate relation in Eq. (28), and consequently the wave power timeseries is developed. The samples are then normalized with the characteristic values $\hat{P}_d$ and $\hat{P}_s$. For illustrative purposes, the wave surface elevation time series were generated for the deep water limit using a JONSWAP wave spectrum model with significant wave height H_s = 4.0m, peak period T_p = 10.0sec, and peakedness factor 0f γ_s =3.3. Regarding these values, $\hat{P}_d = 78.89$ KW/m, that is the mean wave power in an ideal narrow-banded spectrum with zero width. In order to reduce the sample size effects, 60 hours of surface wave elevation (about 26,000 waves) were generated with sample rate of 5Hz employing uniformly distributed random phase in the range of (0, 2π).

In Fig. 6, the wave power sample PDF and CDF estimated from Kernel probability distribution estimation method are compared to those of Rayleigh-Stokes model with parameters estimated from MoM and MoLM. As can be seen in this figure, the simplified Rayleigh-Stokes model is reasonably accurate in capturing the variability of sample wave power. Both MoM and MoLM converge to almost identical estimates, as the sample size is considerably large. It is observed that the semi-empirical model over-predicts the PDF of small wave powers $p_d < 0.5$. This over-prediction is mainly due to the narrow-banded assumption in the Rayleigh-Stokes model that is not fully valid in the studied case.

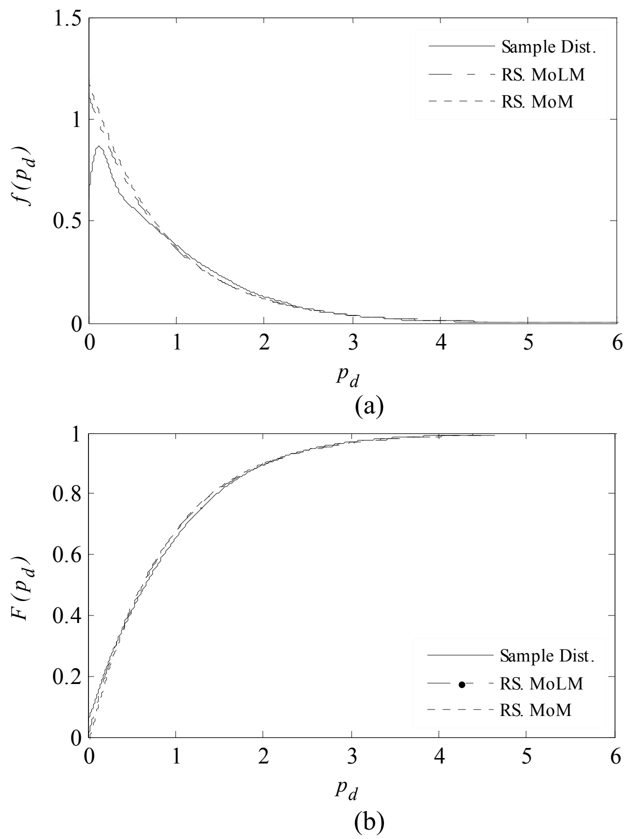

Figure 6- probability distributions of wave power (a) PDF, (b) CDF.

SUMMARY AND CONCLUSION

In this study, semi-empirical models are introduced as an alternative to theoretical and empirical probability distributions. Similar to theoretical models, the structural form of a semi-empirical model is developed from a mathematical model and the model parameters are estimated empirically utilizing sample data. As compared to theoretical models, semi-empirical models have more flexibility in capturing the probability distribution of data and have a wider range of application. The main advantage of semi-empirical model to commonly used empirical models is that the model parameters of semi-empirical models provide some physical insight about the process.

In this study, the focus was on the three-parameter Rayleigh-Stokes model of non-linear random variables developed from quadratic transformation of linear random variable. The unknown model parameters were estimated from two moment based parameters estimation methods, i.e. method of moments and method of L-moments. For this purpose, the explicit relation between the model parameters and the sample statistics were derived. These relations were then used to estimate the model parameters. The semi-empirical Rayleigh-Stokes model can be applied to estimate the probability distribution of a wide range of non-linear random variables in the fields of wave mechanics and wave-structure interaction.

Two examples were utilized to illustrate the application of semi-empirical Rayleigh-Stokes model. In the first example, the three-parameter Rayleigh-Stokes model was used to capture the probability distribution of measured wave crests in the area close to a mini-TLP. The interaction between the incident waves and the diffracted and radiated wave from the floating structure results in a complex wave field around the structure. The results of previous studies indicated that the one-parameter theoretical Rayleigh-Stokes model could not adequately model the probability distribution of disturbed wave crests. However, it was shown here that the three-parameter Rayleigh-Stokes model was successful in capturing the complex nature of wave crests interacting with the structure. It was also observed that the error of the extreme estimates of the semi-empirical Rayleigh-Stokes was reasonably small and therefore the model can be applied for estimation of extreme statistics of weakly non-linear random variables.

The second example focused on the random viability of ocean wave power. It was shown theoretically that a simplified Rayleigh-Stokes model with no linear term could be used to model the probability distribution of wave power of narrow-banded waves. The semi-empirical model was used to estimate the probability distribution of wave samples obtained from simulated wave elevation timeseries. Although the simulated waves did not represent a narrow-banded process, it was observed that the two-parameter Rayleigh-Stokes model is reasonably accurate in representing the probability distribution of wave power sample.

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