Proceedings of OMAE09
28th International Conference on Ocean, Offshore and Arctic Engineering
May 31 - June 5, 2009, Honolulu, Hawaii

OMAE2009-80212

LINEARIZATION OF QUADRATIC DRAG TO ESTIMATE CALM BUOY PITCH MOTION IN FREQUENCY-DOMAIN AND EXPERIMENTAL VALIDATION

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ABSTRACT

Estimate of the pitch motion of an oil offloading Catenary Anchor Leg Mooring (CALM) buoy is presented. Linearization of the quadratic drag/damping term is investigated by the frequency-domain analysis. The radiation problem is solved to estimate the added mass and radiation damping coefficients, and the diffraction problem is solved for the linear wave exciting loading. The equation of motion is solved by considering the linearized nonlinear drag/damping.

The pitch motion response is evaluated at each wave frequency by iterative and various linearization methods of the nonlinear drag term. Comparisons between the linear and non-linear damping effects are presented. Time-domain simulations of the buoy pitch motion were also compared with results from the frequency-domain analysis. Various linearization methods resulted in good estimate of the peak pitch response. However, only the stochastic linearization method shows a good agreement for the period range of the incident wave where typical pitch response estimate has not been correctly estimated.

INTRODUCTION

Understanding and predicting the pitch motion of an oil offloading Catenary Anchor Leg Mooring (CALM) buoy is of practical interest for the mooring system design. Experimental model tests and numerical simulations in the time-and frequency-domains have been used to describe and predict the CALM buoy behavior under environmental loadings.

However, nonlinearities due to buoy skin friction, i.e. viscous pitch damping, result in inconsistencies between the experimental and numerical pitch response. Ryu *et al.* (2006) used the Morison equation to accurately predict the buoy pitch motion in the time-domain. The viscous effects were considered by employing viscous drag elements in a fully coupled time-domain analysis and a diffraction model of the buoy.

In this paper, the viscous drag effect and the natural period of the pitch motion are estimated analytically. The analytical results are compared with experimental model test results. The linear viscous drag coefficient calculated analytically is used to solve the equation of motion for the buoy in the frequency-domain. However, the equation of motion includes nonlinear term of viscous drag. To linearize the nonlinear term, quadratic, cubic, and stochastic linearization methods are employed, and the numerical calculation in the frequency-domain were carried out. The quadratic and cubic linearizations can be described as iterative linearizations, since the solution is obtained by iteration over the equation of motion in the frequency-domain.

Comparisons between the numerical calculation results in the frequency-domain, the time-domain and the experimental model tests are presented. The pitch motion RAO evaluation is assessed. Fig. 1 summarizes the steps used to calculate and validate the pitch motion response.

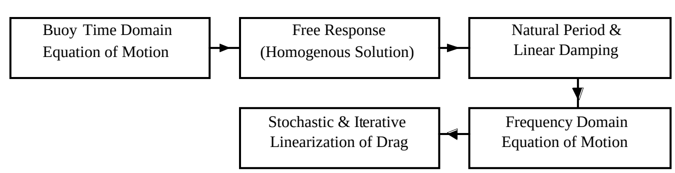

Figure. 1 Steps to calculate the buoy pitch motion response.

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BUOY MODEL

The model tests were designed to provide data for the response of the buoy with no mooring influence so as to allow direct validation of the buoy hull model response. Model tests were conducted in the Offshore Engineering Basin at the Institute for Marine Dynamics in Canada (Ryu *et al*., 2006).

The buoy hull for the test is modeled at a scale of 1:35.6. For the horizontal soft mooring tests the buoy was ballasted to have a free-floating draft of 5.65 meters. The model is fitted with a skirt which has 18 holes. Mooring lines were terminated at load cells to measure the mooring tension at the skirt. All instrument cables were routed out of the buoy model through a suspended umbilical cable. The buoy particulars are listed in Table 1.

Table 1. Buoy model particulars.

	Unit	Mooring
Model Test Scale		35.6
Water Depth	m	106.8
Buoy Hull Diameter	m	17.0
Skirt Diameter	m	21.0
Buoy Height	m	7.65
Draft	m	5.65
Weight in Air	ton	1293.2
KG	m	3.84
Buoy Total Rxx	m	3.82
Buoy Total Ryy	m	3.82
Fairlead Radius	m	9.50
No. of Mooring Legs		4

MODEL BASIN

The tank is 75m long by 32m wide with a variable water depth of up to 3m as shown in Fig. 2. The wavemakers consist of 168 rectangular panels across the front of the tank and along the side in an “L” formation.

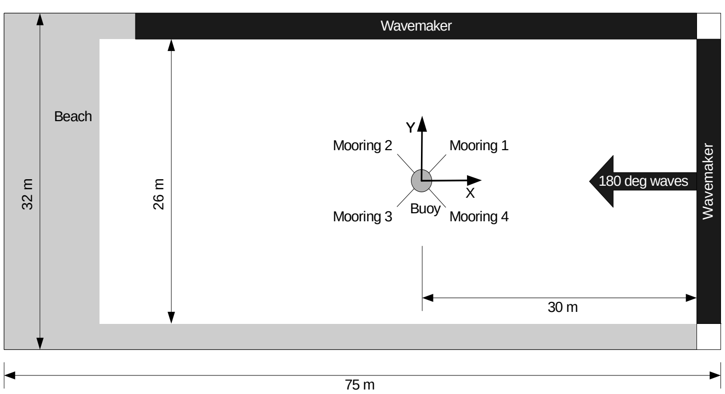

Figure. 2 Plan view of the experimental configuration of buoy.

MOORING CONFIGURATION

The mooring configuration was designed to investigate the motions of a freely floating buoy with minimal influence of the mooring system. The horizontal mooring system consisted of four lines with soft springs that maintained the buoy at the desired location but had minimal feedback to the wave frequency motions. The mooring system was further simplified to reduce influence on the buoy response. Details of the mooring system are provided in Table 2.

Table 2. Particulars of the soft mooring configuration.

	Unit	Mooring
Length	m	350
Wet Weight	kg/m	NA
Diameter	mm	NA
EA	metric tons	180
Pretension	metric tons	22
Fairlead Angle	deg	0

EQUATION OF MOTION

A buoy in waves oscillates in all six degrees of freedom (surge, sway, heave, roll, pitch, and yaw) due to wave exciting loads. The equation of motion of an oscillating floating buoy is studied using the forced vibration of an oscillating mass-spring-damper system. In a matrix format, the equation of motion in the time-domain can be written as follows:

$$M\{\ddot{x}\} + C\{\dot{x}\} + K\{x\} = \{F\} \tag{1}$$

where $M = (M_i + M_a)$ is the mass matrix which consists of hull inertia (M_i) and added mass (M_a), $C = (C_r + C_v)$ is the linear damping matrix which consists of radiation damping (C_r) and (C_v) viscous damping (skin friction), $K = (K_{hydrostatic} + K_{mooring})$ is the stiffness matrix which consists of hydrostatic stiffness $K_{hydrostatic}$ and mooring stiffness $K_{mooring}$, $\{F\}$ exciting wave-induced (diffraction) force vector, and $\{x\}$ is the displacement vector with its time derivatives noted by a dot.

The radiation problem is solved to estimate the added mass M_a and radiation damping C_r coefficients, and the diffraction problem is solved for the linear wave exciting loading $\{F\}$. Additionally, the hydrostatic problem is solved, to determine $K_{hydrostatic}$. The hydrodynamic wave-body interaction program WAMIT was used to solve the buoy radiation/diffraction problem. The following quantities were evaluated by WAMIT:

- Hydrostatic coefficients
- Added-mass and damping coefficients for all modes and periods
- Wave exciting forces and moments
- Motion amplitudes and phases for the freely-floating buoy

Figs. 3 through 5 show the added mass, radiation damping, and wave exciting load of the buoy pitch mode calculated by WAMIT.

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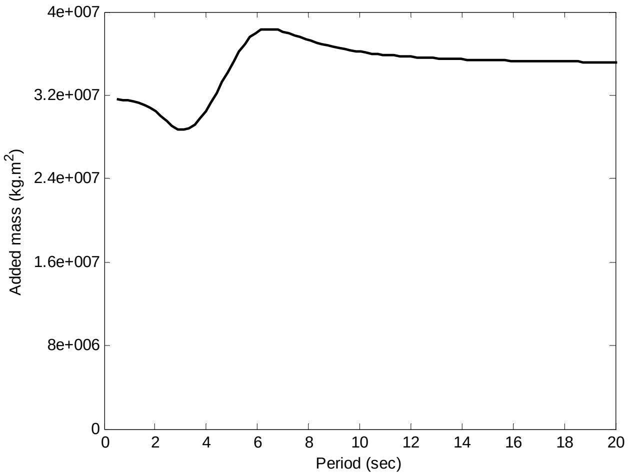

Figure 3: Pitch added mass from the radiation solution.

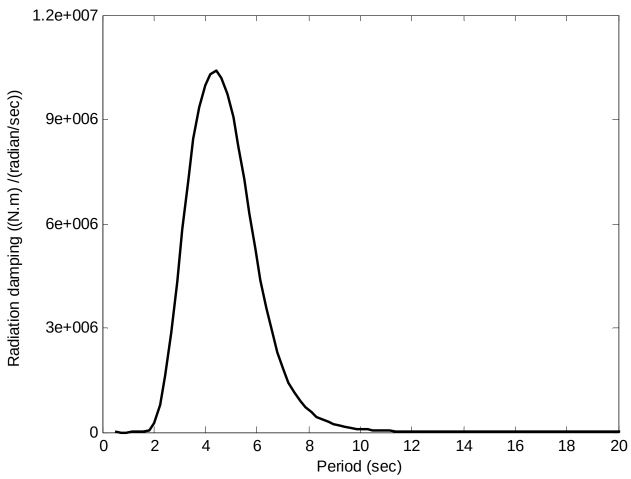

Figure 4: Pitch radiation damping from the radiation solution.

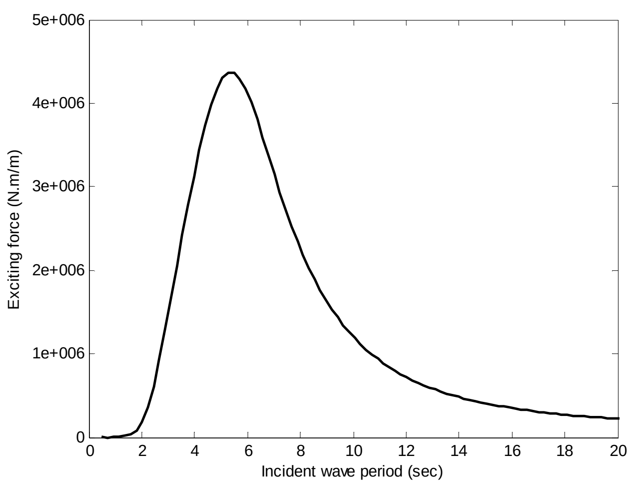

Figure 5: Pitch exciting load from the diffraction solution.

FREQUENCY-DOMAIN SOLUTION

Assuming a harmonic solution to the equation of motion in the time-domain and substituting the solution in Eq. 1, the resulting equation of motion in the frequency-domain becomes:

$$X(\omega) = \frac{F(\omega)}{-\omega^2 M - i\omega C + K} \tag{2}$$

where ω is the frequency of oscillation defined as $\omega = 2\pi/\tau$ and τ is the wave period. The solution to Eq. 2 for all buoy six degrees of freedom (6-DOF) is obtained by dividing the output (wave frequency force) over the input (system characteristics) for each wave frequency of oscillation.

ITERATIVE LINEARIZATION

A nonlinear drag term $C\dot{x}|\dot{x}|$ is introduced to the equation of motion as shown in Eq. 3:

$$M\ddot{x} + C\dot{x}|\dot{x}| + Kx = F \tag{3}$$

Chakrabarti (2001) described a linearization method based on the order of the polynomial expanding the nonlinear drag term. Eq. 6 shows a third order expansion of the nonlinear drag:

$$C\dot{x}|\dot{x}| = C_1\dot{x} + \frac{8}{3\pi}C_2\omega\dot{x}|\dot{x}| + \frac{3}{4}C_3\omega^2\dot{x}^3 \tag{4}$$

in which the first term is linear, the second and the third terms represent the quadratic and the cubic drag respectively.

$$X(\omega) = \frac{F(\omega)}{-\omega^2 M - i\omega\left(C_r + C_v^*\right) + K} \tag{5}$$

where C_v^* is nonlinear drag. Thus the equation of motion in the frequency-domain is solved by substituting Eq. 4 in Eq. 5. For the linear damping case the response is obtained directly by substituting the Morrison drag term as the viscous damping $\left(C_v^*|\dot{x}| = \frac{1}{2}\rho A\right)$ in the equation of motion, where ρ is the water density and A is the buoy projected cross sectional area.

$$X(\omega) = \frac{F(\omega)}{-\omega^2 M - i\omega\left(C_r + \frac{1}{2}\rho A\right)|\dot{x}| + K} \tag{6}$$

For the quadratic damping case the response is obtained by using the following iteration:

$$\begin{aligned} X^0 &= 0, C_v^* = \frac{8}{3\pi}C_v\left|x^n\right| \\ X^{n+1} &= \frac{F(\omega)}{-\omega^2 M - \left(C_r + C_v^*\right)i\omega + K} \\ \left|X^{n+1} - X^n\right| &\le \varepsilon \end{aligned} \tag{7}$$

where n is the number of iterations and ε is the error tolerance. The same iterative scheme is used for the cubic damping:

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$$X^0 = 0, C_v^* = \frac{8}{3\pi}C_v\left|x^n\right| + \frac{3}{4}C_v\,\omega\left(x^n\right)^2$$
$$X^{n+1} = \frac{F(\omega)}{-\omega^2 M - \left(C_r + C_v^*\right)i\omega + K} \qquad (8)$$
$$\left|X^{n+1} - X^n\right| \leq \varepsilon$$

STOCHASTIC LINEARIZATION

Chakrabarti (2002) discussed a stochastic linearization of the nonlinear drag term in the form:

$$C\dot{x}\left|\dot{x}\right| = C\sqrt{\frac{8}{\pi}}\sigma_{\dot{x}}\dot{x} \qquad (9)$$

where $\sigma_{\dot{x}}$ is the standard deviation of the pitch motion oscillating velocity. Thus the solution based on the stochastic linearization of the drag term in the frequency-domain results in Eq. 10:

$$X(\omega) = \frac{F(\omega)}{-\omega^2 M - \left(C\sqrt{\frac{8}{\pi}}\sigma_{\dot{x}}\dot{x}\right) + K} \qquad (10)$$

ANALYTICAL SOLUTION OF PITCH FREE DECAY

The free (transient) response is calculated analytically by solving an initial value problem of the homogenous differential equation of problem described in Eq. 11:

$$M\ddot{x} + C\dot{x} + Kx = 0$$
$$x(0) = X_o, \dot{x}(0) = V_o \qquad (11)$$

The pitch motion transient solution takes the form of an under damped mass-spring-damper system periodically decaying in a manner analogous to the model test free decay response. Eq. 12 describes the pitch motion decay in a closed form:

$$x_h(t) = X_o e^{-\zeta\omega_o t}\cos\left(\sqrt{1-\zeta^2}\,\omega_o t + \varphi\right) \qquad (12)$$

where X_o and φ are obtained from initial conditions, ω_o is the natural frequency, ω the forcing wave frequency, ζ the damping ratio defined as $\zeta = C/2M\omega_o$ and t time.

RESULTS AND DISCUSSION

For the freely floating buoy, the pitch free decay comparison between the experimental result, analytical solution and the time-domain simulation is presented in Fig. 6. Using a closed form analytical solution (Eq. 11) to determine the free response (transient solution) to the buoy equation of motion, the natural period and drag damping for the pitch mode are estimated. The calculated drag is used in evaluating the pitch response for the cases of quadratic, cubic and stochastic viscous drag.

It is noted that near the natural frequency (7 sec), the pitch motion RAO is out of phase with the pitch load RAO as shown in Fig. 7.

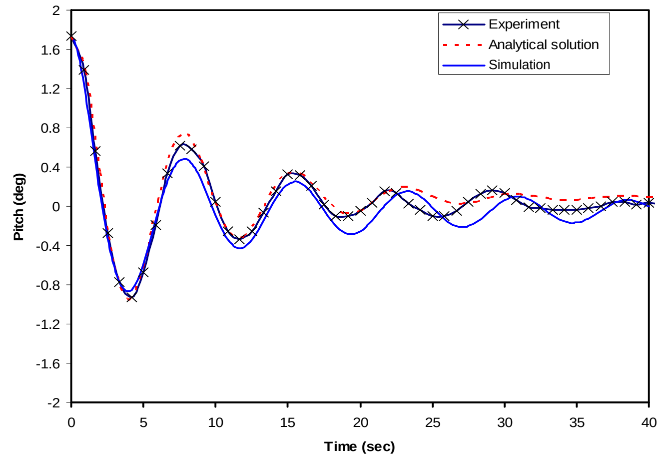

Figure 6: Pitch free decay comparison between experimental result, analytical solution, and time-domain simulation.

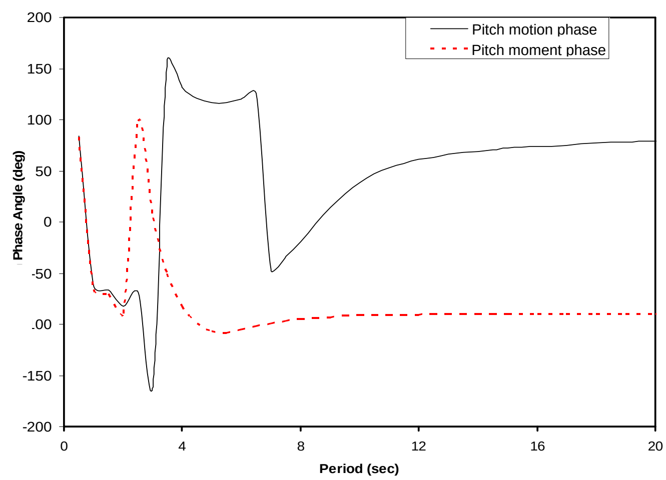

Figure 7: Phase comparison for buoy motion and wave exciting moment.

COMPARISONS OF CALCULATION RESULTS

The frequency-domain solutions to the freely floating buoy motion using iterative Eqs. 7 and 8, and stochastic Eq. 10 methods are presented in Fig. 8. The quadratic and cubic linearizations of the nonlinear drag results in a deep valley near the pitch natural period as the linear numerical results in the frequency-domain. The stochastic linearization of the nonlinear drag provides a good match to the experimental result and the time-domain simulation as shown in Fig. 8.

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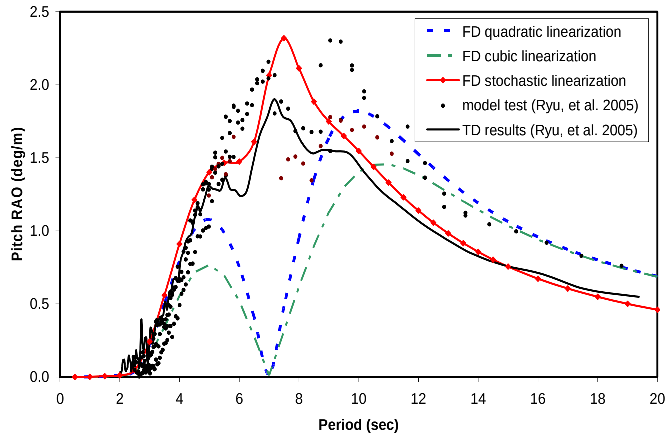

Figure 8: Pitch motion RAO comparison: model test, frequency-domain and time-domain.

CONCLUSIONS

Linearization of the quadratic drag/damping term was addressed for the pitch response of a CALM buoy system. Various linearization methods were implemented and each result was compared. The frequency-domain solution to a CALM buoy equation of motion was considered and the time-domain results obtained from previous research results were also compared.

Numerical results obtained by using different linearized methods showed good agreement in the estimate for the maximum pitch motion. However, the pitch motion estimate based on the iterative (i.e. quadratic and cubic) linearization of the nonlinear term still shows the deep valley around 6 to 8 seconds of incident wave period as the typical linear numerical results in the frequency-domain.

It is noted that the stochastic linearization shows an excellent estimate for the same range of incident wave periods, and it is found that a good agreement exists between experimental results and the proposed linearized drag term approach in the frequency-domain.

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LIST OF SYMBOLS

A	Projected cross sectional area
C	Damping matrix
C_r	Frequency dependent damping matrix
C_v	Drag/skin friction
C_v^*	Nonlinear drag
F	Exciting force
K	Stiffness matrix
$K_{hydrostatic}$	Hydrostatic stiffness
$K_{mooring}$	Buoy mooring stiffness
M	Mass matrix
M_a	Frequency dependent added mass matrix
M_i	Rigid buoy inertia matrix
X_o	Initial displacement
V_o	Initial velocity
t	Time
x	Displacement
x&	Velocity
x&&	Acceleration
x_h	Transient response
e	Error tolerance
r	Water density
z	Damping ratio
$s_{\overset{o}{x}}$	Velocity standard deviation
j	Phase angle
w	Wave frequency
w_o	Natural frequency

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