

OMAE2008-57632

## FREQUENCY-DOMAIN CALCULATIONS OF MOORED VESSEL MOTION INCLUDING LOW FREQUENCY EFFECT

**C. Le Cunff**

Principia Houston, Areva Group  
5177 Richmond Av., Houston, Texas 77056

**Sam Ryu**

SOFEC, Inc  
14741 Yorktown Plaza Dr., Houston, Texas 77040

**Jean-Michel Heurtier**

Principia R.D., Areva Group  
ZI Athelia I - Voie Ariane, 13705 La Ciotat, France

**Arun S. Duggal**

SOFEC, Inc  
14741 Yorktown Plaza Dr., Houston, Texas 77040

### ABSTRACT

Frequency-domain analysis can be used to evaluate the motions of the FPSO with its mooring and riser. The main assumption of the frequency-domain analysis is that the coupling is essentially linear. Calculations are performed taking into account first order wave loads on the floating structure. Added mass and radiation damping terms are frequency dependent, and can be easily considered in this formulation. The major non-linearity comes from the drag force both on lines and the floating structure. Linearization of the non-linear drag force acting on the lines is applied.

The calculations can be extended to derive the low frequency motion of the floating structure. Second order low frequency quadratic transfer function is computed with a diffraction/radiation method. Given a wave spectrum, the second order force spectrum can then be derived. At the same time frequency-domain analysis is used to derive the low frequency motion and wave frequency motion of the floating system.

As an example case, an FPSO is employed. Comparison is performed with time domain simulation to show the robustness of the frequency-domain analysis. Some calculations are also performed with either low frequency terms only or wave frequency terms only in order to check the effect of modeling low and wave frequency terms, separately. In the case study it is found that the low frequency motion is reduced by the wave

frequency motion while the wave frequency motion is not affected by the low frequency motion.

### INTRODUCTION

A frequency-domain approach was presented in Le Cunff *et al.* (2007) for computing coupled floater/mooring system response to first-order wave excitation. The applied calculations are based on the following two steps:

Step 1: A frequency-domain analysis of the floater alone to derive its hydrodynamic characteristics in terms of added mass, radiation damping and wave excitation.

Step 2: A frequency-domain integration of the equation of motion where mooring and risers are described typically with a finite element model while the floater is described as a specific node of the system, its inertia, damping, stiffness and hydrodynamic terms being provided in Step 1.

To obtain the second order motion, the quadratic transfer functions are also evaluated in the first step. The functions are then used to derive the force spectrum on the floater by convolution with the wave spectrum.

The present paper consists of the following two main sections:

- Formulation of the frequency-domain method; and
- Comparison between the frequency- and time-domain calculation results.

## FORMULATION OF FREQUENCY-DOMAIN VERSUS TIME-DOMAIN APPROACH

### Equation of Motion

The equation of motion for the first order wave force is detailed in Ricbourg (2005), and Le Cunff *et al.* (2007). The equation of translational motion can be written as follows:

$$\langle K \rangle x + \langle B \rangle \dot{x} + \langle M \rangle \ddot{x} = F \quad (1)$$

where  $\langle K \rangle$  is the stiffness matrix,  $\langle B \rangle$  damping matrix,  $\langle M \rangle$  mass matrix,  $F$  exciting force vector, and  $x$  translation vector with its time derivatives noted by a dot.

All terms in Eq. 1 can be time dependant. The assumption of the frequency-domain analysis is that they can be represented by their static equilibrium value plus a frequency dependant component. The number of discrete frequency required to accurately represent the solution will depend on the specific system. The main restriction is that the system can be linearized about its static equilibrium.

When dealing with an offloading system, specific hydrodynamic frequency dependant terms have to be taken into account on the floating body's degrees of freedom. Therefore the following contribution will be included respectively in the matrix  $\langle M \rangle$ ,  $\langle B \rangle$ , and  $\langle K \rangle$  of Eq. 1; added mass  $\langle M_a \rangle$ , radiation damping and quadratic damping,  $\langle B_{rad} \rangle$  and  $\langle B_{quad} \rangle$ , hydrostatic stiffness  $\langle K_{hyd} \rangle$ . For the forcing termes  $\langle F \rangle$ , there are four contributions to take into account; 1<sup>st</sup> and 2<sup>nd</sup> order wave force  $F_{wave}$ , mooring force acting on the floating body  $F_{mooring}$ , current load  $F_{current}$ , and wind load  $F_{wind}$ .

A diffraction/radiation program can be used to compute the first-order wave exciting loads, added mass, and radiation damping.

For the frequency-domain analysis, the time-dependant solution  $x$  is written as the sum of a static component and a frequency component. The latter is based on a Fourier representation of the time-dependant part of the solution. The latter itself is divided into imposed motion response in order to satisfy the kinematic boundary conditions and a component which is solution of the floating system:

$$x(t) = X_{stat} + \sum_{i=1}^{N_{freq}} \left\{ \text{Re} \left\{ \left[ \sum_{imp=1}^{N_{imp}} a_{imp}(\omega) x_{imp} + x_i(\omega) \right] e^{-j\omega t} \right\} \right\} \quad (2)$$

In Eq. (2) the static part is obtained by computing the static equilibrium

$$\langle K_{stat} \rangle X_{stat} = F_{stat} \quad (3)$$

For the imposed motion,  $a_{imp}$  is the complex coefficient associated with the motion, and  $x_{imp}$  is obtained by solving

$$\langle K_{stat} \rangle x_{imp} = 0 \quad (4)$$

In effect, the stiffness matrix is the sum of a static and a frequency dependent matrix. The damping and mass matrices

are also decomposed into two matrices: one frequency independent and the other frequency dependent. The former can be computed once for all at the beginning of the frequency-domain calculations.

Finally, all terms have to be linearized to be expressed as a complex coefficient multiplied by  $e^{-j\omega t}$ , and the equation of motion (Eq. (1)) becomes

$$\left[ \langle K(\omega) \rangle - j\omega \langle B + B_a(\omega) \rangle - \omega^2 \langle M + M_a(\omega) \rangle \right] \{X(\omega)\} = \{F(\omega)\} \quad (5)$$

A finite element software, DeepLines (2004), was used to solve Eq. (5).

### Linearization of Nonlinear Term

The velocity quadratic drag force on the lines and floating bodies introduces a nonlinear term in the equation of motion. This drag term is proportional to the velocity squared. To conduct a frequency-domain analysis, it is therefore necessary to introduce a linearization as indicated in Eq. (6).

$$\|v_{rel}\| v_{rel} \approx \Omega (v_{rel}) v_{rel} \quad (6)$$

The linearization coefficients are detailed in Geld *et al.* (1968) and Kroliwowski *et al.* (1980). For irregular wave the linearization coefficient is given by  $\Omega = \sqrt{2/\pi} 2\sigma$ , where  $\sigma$  is the standard deviation of the relative velocity.

### Waves Exciting Forces

Both first- and second-order wave frequency (WF) force spectra are derived. For a given wave spectrum, a certain number of frequency ( $nh$ ) is chosen to represent the spectrum. For each frequency the wave amplitude is computed, using the following equation,

$$A(\omega) = \sqrt{2\Delta\omega S(\omega)}, \quad (7)$$

where  $A$  is the amplitude,  $S$  the spectrum and  $\Delta\omega$  the difference between the two consecutive frequencies.

The wave velocity is then derived, as well as the relative fluid/structure velocity. For the relatively slender bodies such as risers and mooring lines the Morison formula can be applied to obtain the wave exciting force on the structure. For large floating structures, the force RAOs computed by a diffraction/radiation method are multiplied by the wave amplitude at the respective frequencies, and the resulting first-order wave force spectrum is obtained. The formulation is used in frequency domain, and time domain with proper consideration of phasing to compute the response of the system.

For the second order low frequency (LF) spectrum, the approach is slightly different between time domain and frequency domain. The wave spectrum provided  $nh$  wave ( $A_i$ ,  $i$

= 1, nh) amplitude. Obviously, there are two types of second order forces, characterized at the sum mode and the difference mode. The first one corresponds to the summation of frequency and leads to high frequency force, and the second one is based on the frequency difference between the two wave components and leads to the low frequency force. Therefore, only the difference mode is of interest in the present study.

Following Molin (2002), calculations of the low frequency force on the floater in time are given by

$$F_{wave}^z = \Re \left( \sum_{l=1}^{nh} \sum_{m=1}^{nh} A_l A_m f^{(2)}(\omega, \omega_m) \exp[-j(\omega - \omega_m)t + j(\theta_l - \theta_m)] \right) \quad (8)$$

with  $\theta_i$  the phase of the  $i^{\text{th}}$  wave component and  $f^{(2)}$  the quadratic transfer function. The wave elevation is expressed in time with the following formula

$$\eta = \Re \left( \sum_{l=1}^{nh} A_l \exp[-j\omega t + j\theta_l] \right) \quad (9)$$

Calculations are typically carried out in time domain over 3 hours with a low frequency motion being in the order of 100s. Since we have  $nh$  components in the wave spectrum, there are  $nh(nh-1)/2$  low frequency components. In frequency domain, a linear system is solved for each frequency on the degrees of freedom of the system. Coupled systems may have a large number of mooring and risers, and it will become time consuming to solve  $O(nh^2)$  linear problems. Furthermore, for the case of moored structure, there is a well defined response of the structure. Indeed, for low frequency, the hydrodynamic characteristics of the structure become frequency independent, and there is a dominant resonant response of the structure on its main axis. Actually, the motions of interest are the surge, sway and yaw of the structure. Therefore the analysis is performed in frequency domain by computing the spectrum of the force over  $nh$  components.

The low frequency force spectrum is given as a function of frequency  $\Omega$  by:

$$S_F^{(2)}(\Omega) = 8 \int_0^{\infty} S(\omega) S(\omega + \Omega) \|f^{(2)}(\omega, \Omega - \omega)\|^2 d\omega \quad (10)$$

As for the wave spectrum (see formula (7)), the amplitude of the force is obtained for each low frequency from the spectrum. To summarize, wave frequency force is used from  $\omega_{inf}$  to  $\omega_{sup}$ . These frequencies are the lower and higher boundary of the wave spectrum. From 0 to  $\omega_{inf}$ , the low frequency force is used. The linearized coefficients previously described apply over all frequencies from 0 to  $\omega_{sup}$ .

Finally, for the quadratic transfer function, it is possible to use the direct calculations of the function or the Newman's approximation. In the latter case, the drift force  $f_d(\omega)$  is computed for all wave frequencies, and the quadratic transfer function is approximated by

$$\|f^{(2)}(\omega, \omega_m)\|^2 = |f_d(\omega)| |f_d(\omega_m)| \quad (11)$$

A radiation/diffraction software, Diodore (2005), can be used to calculate the drift force.

The second order wave potential is also considered in some of the calculations. The wave elevation is represented by Eq.9, the incident wave associated with the difference mode leads to a low frequency force contribution. Assuming that an (x, y, z) reference frame is attached to the body with z the upward vertical direction, and that  $\beta$  is the angle between the wave direction and the x-axis, the transfer function of the force for infinite depth is given by (Molin, 2002):

$$\|f_x^{(2)}(\omega, \omega_m)\| = \rho(1 + C_{m_x}) V \frac{\omega \omega_m (\omega - \omega_m) (k - k_m)}{g(k - k_m) [h[(k - k_m)h] - (\omega - \omega_m)^2]} \times \frac{ch[(k - k_x)(z + h)]}{ch[(k - k_x)h]} \cos \beta, \quad (12)$$

in the x-direction where

$\rho$ : fluid density

$V$ : immersed volume

$C_{m_x}$ : asymptotic added mass at low frequency

$k_i$ : wavenumber associated with the  $i^{\text{st}}$  wave component.

The function in the y-direction is obtained similarly by replacing  $C_{m_x}$  by  $C_{m_y}$  (asymptotic added mass in the y-direction) and  $\cos \beta$  by  $\sin \beta$ . The force is applied at the hydrodynamic center of the floating structure.

## Wind Forces

The wind force is computed if the wind is expressed as a wind spectrum with a constant direction. Frequency-domain analysis cannot handle changes of the wind direction typical of squall wind for instance. To define the wind force on a floating structure, the following parameters are used:

- a reference surface,  $S_r$ ,
- location of the center of the wind force with respect to the center of gravity of the floating structure,
- wind coefficients ( $C_{vx}$ ,  $C_{vy}$ ,  $C_{vz}$ ) as a function of the direction of the structure relative velocity in air with respect to the main axis of the floating structure.

As the wave elevation is derived from the wave spectrum, the wind speed is derived from the wind spectrum as follows

$$V_{wind} = \Re \left( \sum_{l=1}^{nh} A_{wl} \exp[-j\omega t + j\theta_{wl}] \right) \quad (13)$$

As shown in Figure 1, the surge force  $F_x$  on the structure due to the wind is then given by

$$F_{wind,x} = \frac{1}{2} \rho_{air} S_r C_{vx} \|V_{wind} - V_{vessel}\|^2, \quad (14)$$

where  $\rho_{air}$  the density of air, and  $V_{vessel}$  the vessel velocity. The direction of the force is imposed by the wind coefficient. In time domain simulation, the force is directly computed and imposed to the structure at each time step.

In frequency domain there are two further points to address. First, due to the relative velocity formulation, the direction of the velocity can change with time. Therefore in time domain the coefficient  $C_{vx}$  is a function of time. In frequency domain, the wind coefficients are kept constant, evaluated after a static analysis of the structure under mean loads. Secondly, the wind spectrum has a mean velocity. In that case, the linearization coefficient as defined in Eq. (6) is computed as follows

$$\Omega = 4\sigma PI\left(\frac{V_{0,wind}}{\sigma}\right) + 2V_{0,wind} PF\left(\frac{V_{0,wind}}{\sigma}\right), \quad (15)$$

where  $V_{0,wind}$  is the mean velocity of the wind,  $\sigma$  the standard deviation of the relative velocity spectrum, and  $PI$ ,  $PF$  are expressed as

$$PI(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), \quad (16)$$

$$PF(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{\xi^2}{2}\right) d\xi.$$

## COMPARISON BETWEEN TIME-DOMAIN AND FREQUENCY-DOMAIN ANALYSES VIA CASE STUDY

### Reference frame conventions for the system

The sign conventions utilized for the analysis of motions and loads in earth-fixed and vessel-fixed local coordinate systems are shown in Fig. 1. Note that plan view angles increase in a counter-clockwise fashion. Both in the analysis and presentation of computer simulation results, wind, wave and current angles refer to the directions towards which these environments propagate in the given earth-fixed analysis coordinate system.

The FPSO moored by a 4 x 3 anchor leg system which was used for the case study is shown in Fig. 2.

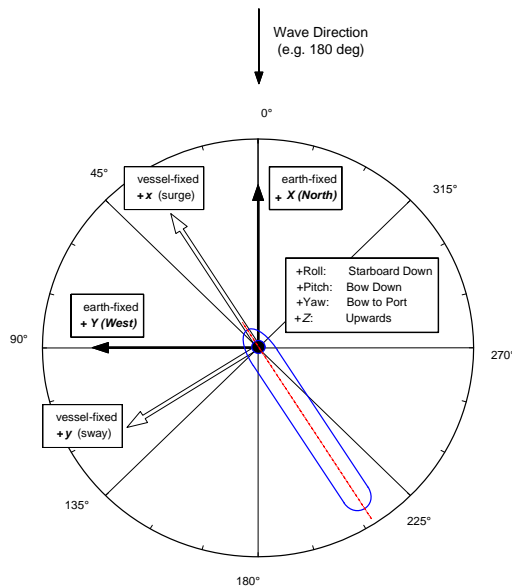


Figure 1: Sign conventions utilized for the analysis of the motions and loads

### Case Study: Moored FPSO

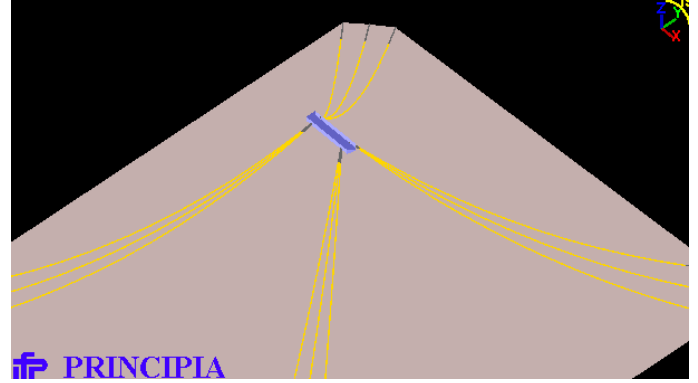


Figure 2: FPSO moored by a 4x3 anchor leg system.

An FPSO in a water depth of 1,300m is used for the case study. Hydrodynamic calculations are performed to derive the first order wave force and the drift force. A JONSWAP spectrum is used to represent the wave with a characteristic wave height of 2.4m and a period of 16.5s. The wave is oriented in the x-direction corresponding to the main axis of the vessel.

In this case study the mooring is modeled as a stiffness matrix. Numerical calculations were performed both in frequency domain and in time domain by taking into account both the wave frequency load and the low frequency load based on the Newman's approximation.

In frequency domain the spectrum is directly found from the norm of the norm at each computed frequency as given in Eq 2. From the displacement amplitude, the spectrum is then computed using Eq. 7. The spectrum is obtained only at the computed frequency. In time domain calculations are performed over 10,800s, and then the time series is extracted as shown in Fig. 3. Both low frequency and high frequency motions are presented in the signal. An FFT of the signal is then performed to extract the response spectrum.

The low frequency motion is computed for surge, sway, and yaw only, while the wave frequency motion is obtained for all six degrees of freedom. In Fig. 4 the FFT of the motion is presented for the three low frequency degrees of freedom. The graph focuses only on the low frequency region. The dominant response has a period of about 300s. The spectrum energy is similar between time domain and frequency-domain calculation.

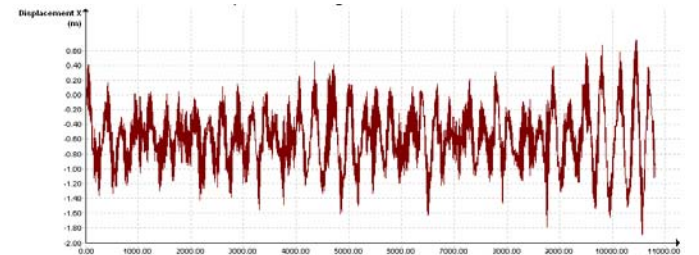


Figure 3: Time series of FPSO surge motion (10,800s). Both low frequency and wave frequency motions can be observed.

The frequency-domain analysis has one dominant peak, while in time domain three peaks are observed. This is due to the fact that in frequency domain, only the frequencies which are excited are post-treated to compute the spectrum. In time domain the FFT does not always correspond exactly to the excited frequencies which are the difference modes between any two wave spectrum components.

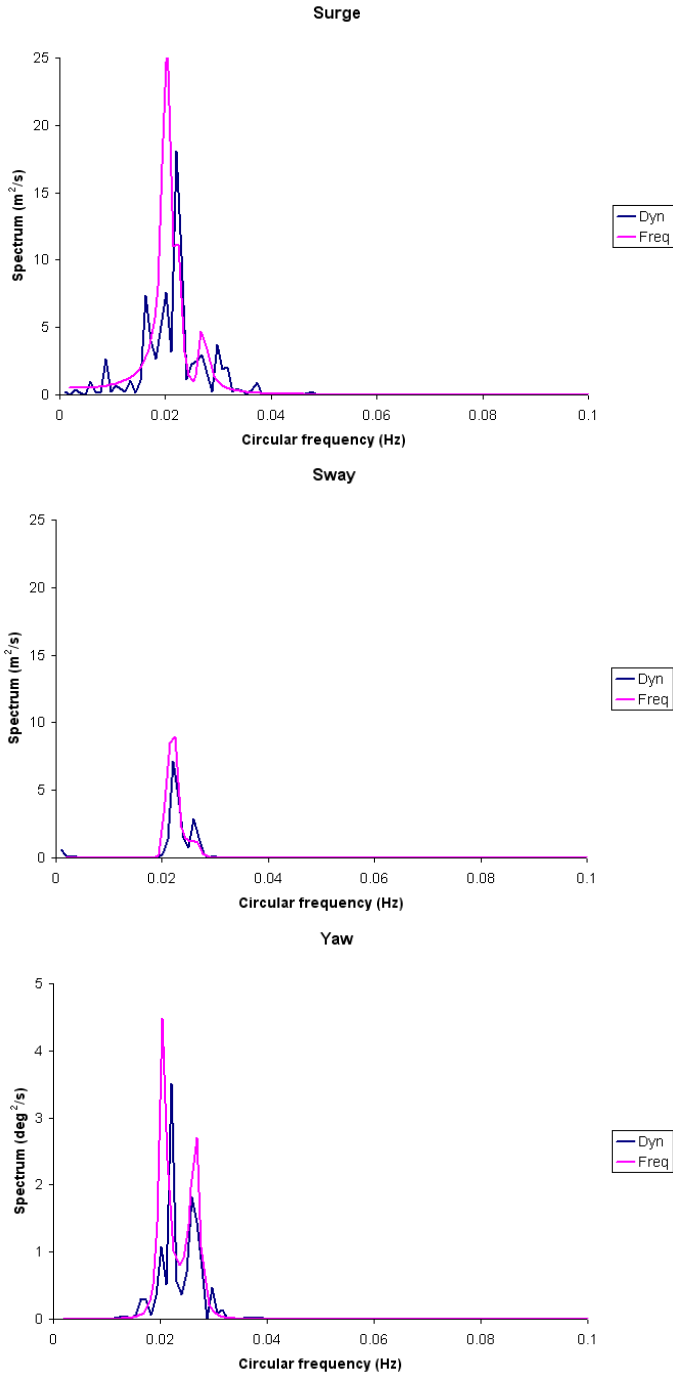


Figure 4: Low frequency response of the FPSO: comparison between time-domain and frequency-domain approach.

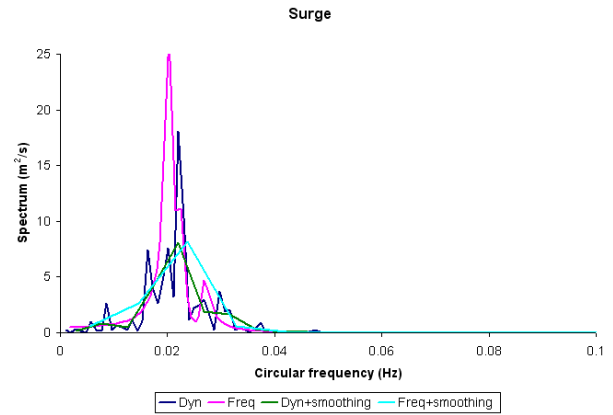


Figure 5: FFT of FPSO surge motion with and without smoothing.

To ensure that the energy is the same for both time- and frequency-domain calculations, a smoothing of the spectrum is performed by averaging the spectrum over several periods but keeping the same total energy. The results for the surge motion are given in Fig. 5, and indicate that the low frequency energy is almost the same for both methods.

### FPSO with Wind Spectrum

A Davenport wind spectrum is chosen to represent the wind with a heading angle of 300 degrees. Both wind and wave spectra are imposed. Calculations are performed both in dynamic and time-domain over one hour. Besides the wind spectrum, the conditions are the same as in the previous section. In particular, the mooring is still represented by a mooring stiffness. Comparison between time and frequency-domain calculations are shown in Figure 6. The spectrum are more energetic than in **Error! Reference source not found.**, due to the extra energy brought by the wind. Again, the frequency-domain analysis is able to capture the vessel motion, even when wave and wind are not collinear.

### FPSO: fully coupled analysis

The comparisons with and without wind are now repeated but in fully coupled analysis. The a 4x3 anchor leg system as described in Figure 2 is now completely modeled together with the hydrodynamic damping on the lines. The results from time and frequency-domain analysis with and without wind are presented in **Error! Reference source not found.** The emphasis is in the surge motion which is the dominant motion. The displacement is well captured by the frequency-domain approach.

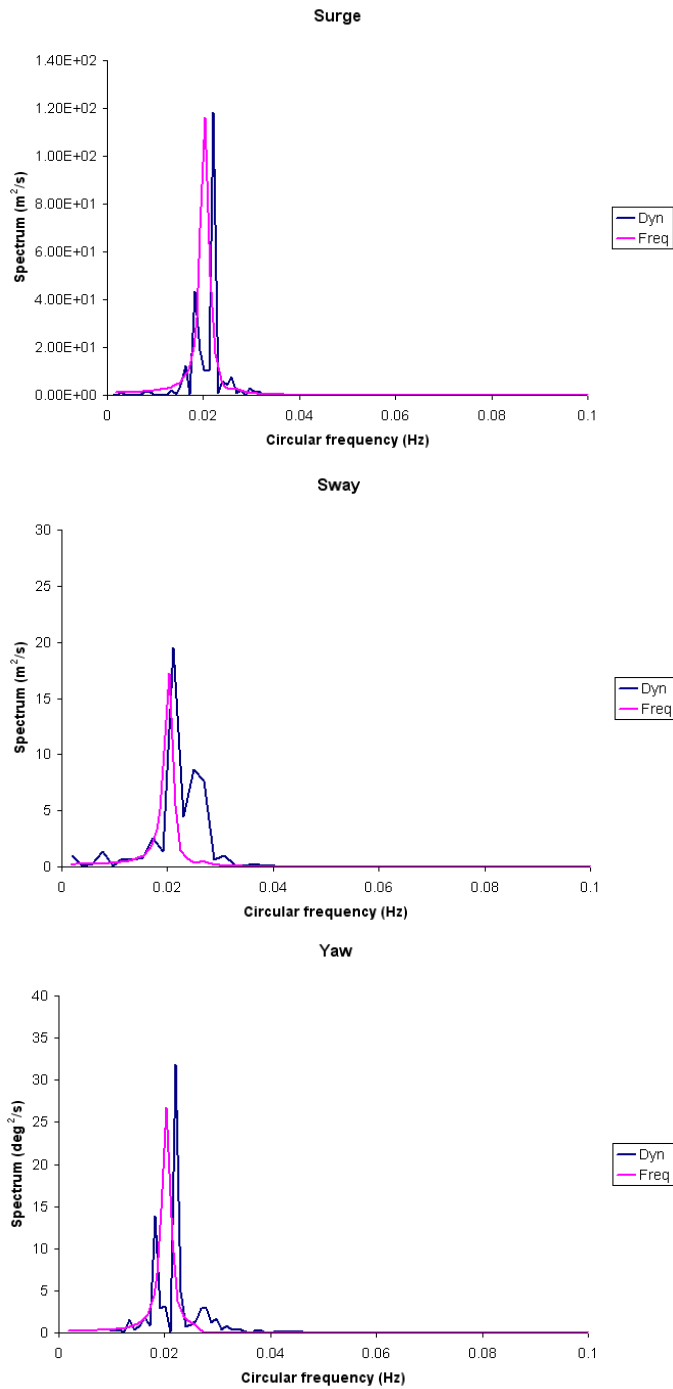


Figure 6: Low frequency response of the FPSO: comparison between time-domain and frequency domain approach with stiffness matrix representing the mooring

## COMMENTS ON MODELING

### Influence of Second Order Wave Potential

Some calculations were performed with and without the consideration of the second order potential incident wave in the low frequency calculations. No second order wave influence

was found. For the FPSO, the force spectrum of the low frequency component (Newman's approximation) was compared to the force due to the second order potential. As shown in Fig. 8, around the low frequency peak response (0.02 Hz), the second order potential force due to incident waves is completely negligible. Therefore no contribution can be expected by taking into account such force.

### Damping Due to WF on LF Motion

A second was performed in coupled analysis with three forcing conditions:

1. first order waves only;
2. low frequency (LF) only; and
3. low frequency and first order wave frequency (WF).

The objective is to check the influence of the damping introduced by first-order waves on the low frequency motion. The results are presented in Fig. 9. As expected adding the low frequency contribution to the first order waves does not modify the response for the wave frequencies. Nonetheless, including the first-order wave motions in the low frequency calculations tends to reduce the low frequency motion. As highlighted in Fig. 9, the peak amplitude at low frequency response is decreased by about a third compared to low frequency forcing only.

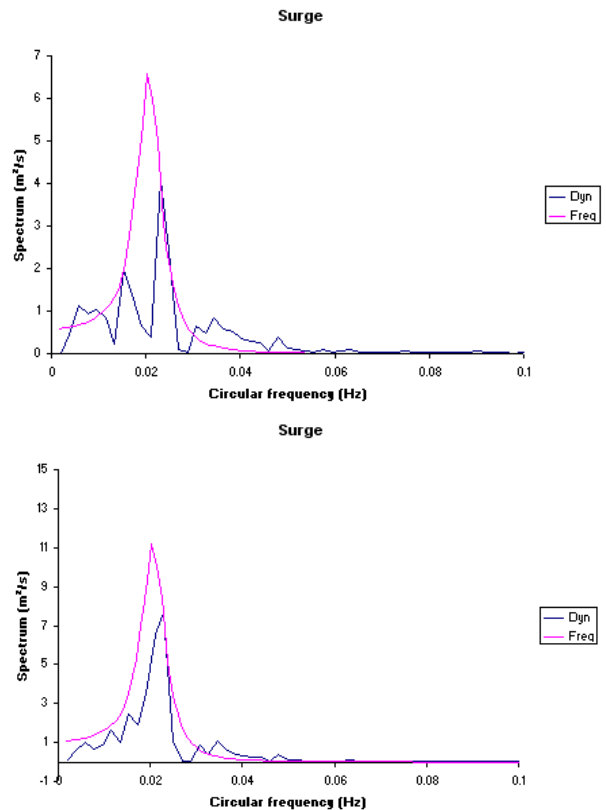


Figure 7: Low frequency response of the FPSO: comparison between time-domain and frequency-domain approach, with wave spectrum (top), and wave+wind spectrum (bottom).

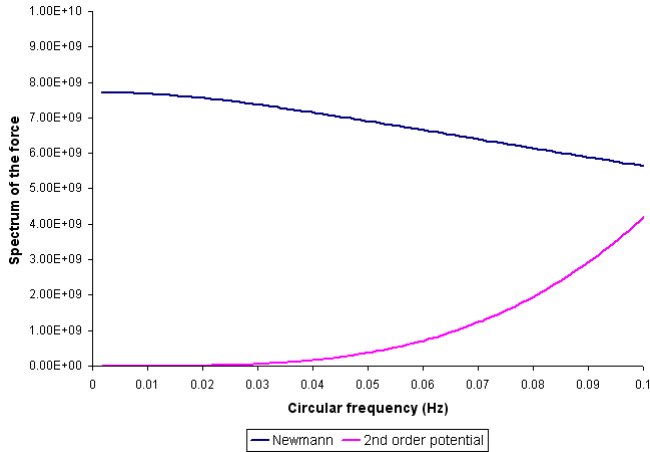


Figure 8: Force spectrum in the surge direction: comparison of the spectrum from Newman's approximation and force from second order wave potential.

### Low Mooring Stiffness

Some good agreement is found in the previous example between time domain and frequency-domain calculations in the prediction of low frequency motion of a moored vessel, even when the wave and wind have different directions. Another interesting example is the influence of a tanker on an export buoy as shown in Fig. 10. The buoy is anchored by its mooring, and two export lines connect the FPSO to the buoy. The buoy itself is not subjected to low frequency motion, but in tanker connected condition the low frequency motion of the tanker induces a low frequency motion of the buoy. The influence of the tanker motion on the deepwater offloading buoy was addressed by Duggal and Ryu (2005).

A comparison of the displacement of the buoy with the wave aligned with the plane of the export line and tanker was made, and the results are presented in Fig. 11. It is found that there is a good agreement between the frequency-domain and time-domain results. The low frequency peak is entirely due to the tanker motion.

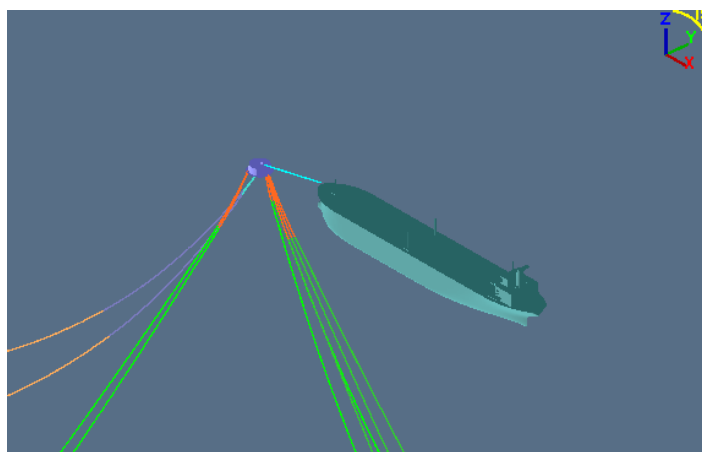


Figure 9: Export buoy with export lines and mooring, in tanker connected conditions.

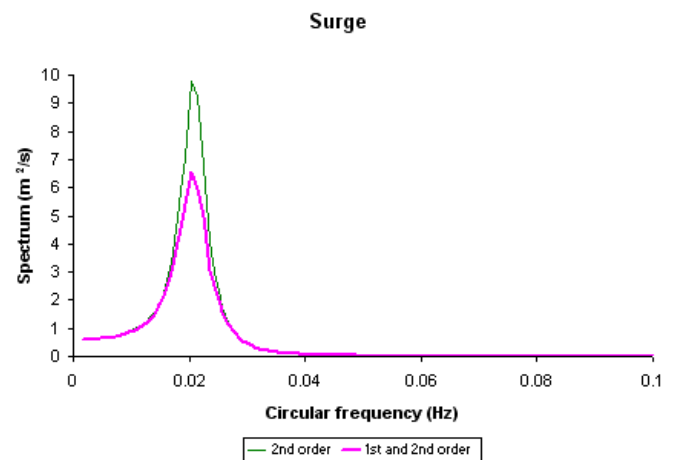
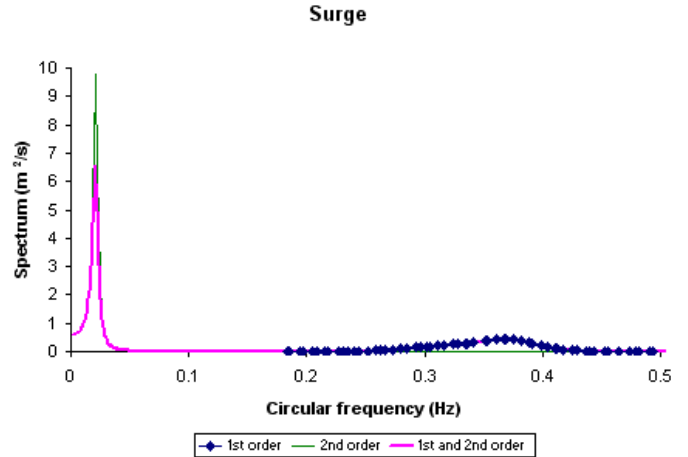


Figure 10: Comparison in frequency domain of the FPSO surge response with coupled analysis by performing first order wave forcing, low frequency forcing only, and both low and first-order wave frequencies. Upper figure presents LW and WF and lower one shows only WF part of the spectrum.

Nonetheless, such an approach is restricted in frequency domain to cases where the environment is in the same direction, and the tanker has a mean stable position. The tanker is connected to the buoy by a hawser, and a constant force is applied to model tug pull. Therefore, the tanker can be unstable depending on several parameters including the tanker length, hawser stiffness and length, etc. In case the tanker is in an unstable condition such as fish-tailing, its yaw angle can vary in time to find stable positions, so obviously in this case the frequency-domain approach can not be used.

### CONCLUSION

A frequency-domain analysis methodology is presented as a tool of motion estimate even including the low frequency response. The classical approach of linearization of the quadratic drag term is addressed to implement the suggested frequency-domain analysis. A case study with a spread moored FPSO shows that there is a good agreement in results between time domain and frequency-domain calculations.

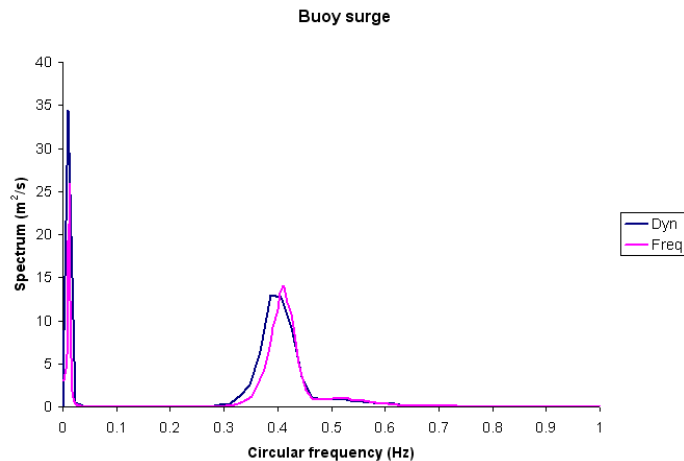


Figure 11: Comparisons of buoy response in time domain and frequency domain with tanker connected.

Duggal, A. and Ryu, S., “The Dynamics of Deepwater Offloading Buoys,” *Fluid Structure Interaction and Moving Boundary Problems*, WIT Press, 2005, pp. 269-278.

It is found that the affect of second order wave potential is not significant. In addition, the suggested frequency-domain analysis was implemented to take into account low frequency motion of a moored FPSO subject to wave and wind. It is shown that good agreement between time domain and frequency -domain calculations for a deepwater case.

Concerning a proper use of the frequency-domain analysis, it is noted that the stiffness should be reasonably linear, that large displacements may not be accurately captured, and that unstable time-varying yaw motion can only be analyzed by using a time-domain analysis. Nonetheless, in the case of low frequency motion of an export buoy induced by the connected tanker, good correlation is still observed provided that the environment and tanker are aligned. Lastly, performing calculations with low and first order frequencies increases the damping compared to low frequency calculations alone.

## REFERENCES

DeepLines Theory Manual, version V4.1, Edited by Principia R.D. 2004.

Diodore Theory manual, version V3.2, Edited by Principia R.D, 2005.

Kroliwkowski, L.P. and Gay, T.A., “An Improved Linearization Technique for Frequency Domain Riser Analysis,” *Proceedings of OTC 1980*.

Geld, A. and Vander Velde W. E, “Multiple-Input Describing Functions and Nonlinear System Design,” *Mc Graw-Hill*, 1968.

Le Cunff, C., Ryu, S., Duggal, A., Ricbourg, C., Heurtier, J.-M., Heyl, C., Liu, and Y., Beauclair, O., “Derivation of CALM Buoy Coupled Motion RAOs in Frequency Domain and Experimental Validation,” *Proceeding ISOPE 2007*.

Molin, B., “Hydrodynamique des Structures Offshore,” *Editions Technip*, June 2002.

Ricbourg, C., “Mise en Place D’une Résolution Fréquentielle dans DeepLines,” *Internal Note, Principia R.D*, 2005.