Derivation of CALM Buoy Coupled Motion RAOs in Frequency Domain and Experimental Validation

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ABSTRACT

Frequency domain analysis is used to solve a complete catenary anchor leg mooring (CALM) buoy system comprised of the buoy, its moorings lines and the export lines. The advantage of such an approach is that it is very fast to run and allows large parametric studies in relatively short times. The underlying assumption of the frequency analysis is that the coupling is essentially linear. Therefore, calculations are performed taking into account first order waves loads on the floater. Added mass and radiation damping terms are frequency dependent and can be easily handled in this formulation. The main source of non-linearity comes from the viscous damping both on lines and buoy. Classical methods are employed to linearize the drag force on the lines and are similarly used for the buoy.

Time domain simulations remain necessary when higher order loads, or drift forces are imposed. But for first order waves, frequency analysis is a powerful and accurate tool to predict buoys motions and evaluate the fatigue life of mooring and export lines submitted to first order excitations. Comparisons are made between numerical simulations and model test results. Good agreement is found between the experimental data and the frequency-domain analysis for the coupled CALM buoy motion response.

KEY WORDS: Deepwater oil offloading buoy; coupled analysis; frequency domain analysis

INTRODUCTION

Deepwater offloading buoys are being extensively used in West Africa to allow efficient loading of spread-moored FPSOs (Ryu *et al*., 2006). Some of the current projects of the offloading buoys include Agbami (Nigeria, 1435m water depth), Akpo (Nigeria, 1285m), Bonga (Nigeria, 1000m), Dalia (Angola, 1341m), Erha (Nigeria, 1190m), Girassol (Angola, 1320m), Greater Plutonio (Angola, 1310m), and Kizomba A & B (Angola, 1200m, 1000m). Compared to other floating systems such as TLP, SPAR, and FPSO, the deepwater offloading buoy system has relatively small displacement and inertia so that the mass, damping, and stiffness of the mooring lines and oil offloading lines (OOLs) can be considerable compared to the inertia, radiation damping, and hydrostatic stiffness of the buoy. Several papers, Huang *et al* (2005), Santala *et al* (2005), Ryu *et al*. (2006) and Ricbourg *et al.* (2006), suggest that the application of Morison elements as an additional drag/damping term improve the motion prediction of the deepwater oil offloading buoy. The motion behavior of this system drives the fatigue damage to the mooring and flowline components, and thus must be estimated accurately to ensure that the system is designed with sufficient fatigue life (Duggal *et al*. 2005).

Due to the strong coupling between lines and floater, the derivation of the RAOs for the floating system coupled with mooring and risers is usually performed in time domain. Duggal *et al*. (2006) found that quadratic drag term contributed pitch motion responses by conducting frequency domain calculations. The objective of the present study is to compare coupled buoy motion obtained in time domain and frequency domain. The premise of this work is based on the assumption, that while the coupling is strong, it is essentially a linear phenomenon.

The calculations are based on two steps:

1. A frequency analysis of the floater alone to derive its hydrodynamic characteristics in terms of added mass, radiation damping and wave excitation.
2. A frequency domain integration of the mechanical equations where mooring and risers are described with typically a FEM model while the floater is described as a specific node of the system, its inertia, damping, stiffness and hydrodynamic terms being provided by step 1.

A first frequency domain methodology was developed by Heurtier (1997) to perform analysis with DeepLines™. The approach was based on solving the mechanical equations on the modal basis of the system. The main drawback of the method is to prevent the use of frequency dependent matrices. The frequency dependence is unfortunately of particular interest for coupled analysis (floating body and its mooring/riser component). Indeed, the characteristics of the floater (added mass and damping) are function of the frequency.

A second approach was therefore proposed by Ricbourg (2006). Since solving a linear problem in the complex domain requires a computer time similar to solving an equivalent system in real domain, solving directly the mechanical equations for one frequency should take the same time as solving the static equilibrium. Therefore, it was proposed to build the matrix in the usual DeepLines™ space (translations and pseudo-rotations at nodes) and to forgo the projection on the modal basis. Obviously, the matrices being built at each given frequency allows the user to enter frequency dependent parameters.

Finally validation of the approach is performed by comparing frequency and time-domain results. Then frequency domain results are compared to measurements of a scaled moored buoy to demonstrate the accuracy of the computational methods being employed to predict the buoy motions.

The present paper is therefore divided in three main sections:

1. Formulation of the frequency domain method;
2. Comparison between the frequency and time domain simulations; and
3. Comparison between frequency domain and model tests.

FREQUENCY DOMAIN THEORETICAL BACKGROUND

Hydrodynamic Calculation via Boundary Element Method

To consider coupling and interactions between a buoy and mooring lines, Eq. (1), the governing equation of the rigid-body dynamics of the buoy, is solved (Ryu *et al*., 2006).

$$\left\{M + M_a(\infty)\right\}\ddot{\vec{X}} + \int_0^\infty R(t-\tau)\dot{\vec{X}}d\tau + K\vec{X} = \vec{F}_d + \vec{F}_w^{(1)} + \vec{F}_w^{(2)} + \vec{F}_{line} \tag{1}$$

where M and M_a represent the mass and added mass matrices, R the retardation function matrix, K the hydrostatic stiffness matrix, $\vec{X}$ the body displacement, $\vec{F}_d$ the drag force, $\vec{F}_w^{(1)}$ and $\vec{F}_w^{(2)}$ the first- and second-order wave loads, $\vec{F}_{line}$ the interface loads from the mooring lines and the OOLs, and the arrow above each variable the column vector. The total velocity potential is given by Eq. (2).

$$\phi = \phi_D + \phi_R \tag{2}$$

where the diffraction potential ϕ_D is defined as the sum of the incident wave potential ϕ_I and the scattered potential ϕ_S .

A diffraction/radiation program based on a boundary element method with constant panels (WAMIT for the experimental cases, DIODORE for the full scale system) is used to calculate the buoy first-order wave exciting loads, the added mass, and the radiation damping based on the previously mentioned equations. The hydrodynamic results of the buoy alone are transferred to DeepLines, a fully-coupled time-domain analysis program, to calculate the buoy motion and mooring/export line motion as described in Eq. (1).

RAOs in Time Domain

In order to obtain the response of a coupled system to wave loading, time domain calculations are performed with either regular or irregular waves. For regular wave, analyses are performed at each period of interest, on a time frame of order of 10 periods excluding transient. In irregular wave, a typical three hours simulation with a wave spectrum is conducted and analysis of the signal is performed to retrieve the phase and amplitude of the motion at each period.

The drawback of these calculations is that it can be quite time consuming when the number of lines increases. Therefore, a simpler and faster method is very useful to conduct parametric studies before using the more complex method. The main objective of this paper is to demonstrate that frequency domain analysis can provide very quickly meaningful results, due to the fact that the coupling mechanism is essentially linear.

Mechanical Equations

The general equation of motion can be written (for the translation degrees of freedom):

$$Kx + B\dot{x} + M\ddot{x} = F \tag{3}$$

where K is the stiffness matrix, B damping matrix, M mass matrix, F loading, and x the value of the translation with its time derivatives noted by a dot.

When dealing with an offloading system, specific hydrodynamic terms have to be taken into account on the buoy's degrees of freedom:

M_a	: added mass,
B_{rad}	: radiation damping
B_{quad}	: quadratic damping
K_{hyd}	: hydrostatic stiffness
F_{Wave}	: 1st and 2nd order wave forces
$F_{mooring}$	: tension imposed on the buoy at the lines attached points..
$F_{current}$	: loads due to the current
F_{wind}	: loads due to the wind

For a frequency domain analysis, the solution X ( ) is written as the sum of a static component and a frequency component. The latter is itself divided into imposed motion response (in order to satisfy the kinematic boundary conditions) and a component which is solution of the mechanical systems:

$$X(t) = X_{stat} + \sum_{i=1}^{Nbfreq}\left\{\mathrm{Re}\left\{\left[\sum_{imp=1}^{Nimp} a_{imp}(\omega_i)x_{imp} + x_f(\omega_i)\right]e^{-j(\omega_i t)}\right\}\right\} \tag{4}$$

In Eq. (4), the static part is obtained by computing the static equilibrium where K_{stat} is the static stiffness matrix and F_{stat} the static forces on the system:

$$K_{stat}x_{stat} = F_{stat} \; . \tag{5}$$

For the imposed motion x_{imp}, a_{imp} is the complex coefficient associated with the motion, and x_{imp} is obtained by solving:

$$K_{stat}x_{imp} = 0 \; . \tag{6}$$

In effect, the stiffness matrix is the sum of a static and a frequency dependent matrix. The damping and mass matrices are also decomposed into two matrices: one frequency independent, one frequency dependent. The former can be computed once and for all, at the beginning of the frequency calculations.

In the frequency domain formulation, the retardation function is not used as in Eq 1. Rather, the added mass and radiation damping value at each frequency are used. Finally, all terms have to be linearized to be expressed as a complex coefficient multiplied by $e^{-j\omega t}$, and the mechanical equations Eq. 3 becomes:

$$\left[K(\omega) - j\omega\left(B + B_a(\omega)\right) - \omega^2\left(M + M_a(\omega)\right)\right]\{X(\omega)\} = \{F(\omega)\} \tag{7}$$

Linearization of Quadratic Damping

For deepwater offloading buoys, tests performed in the JIP CALM buoy (2005) confirmed that most of the damping contribution is of quadratic nature and the linear component is mainly related to wave radiation. Thus, the recommended practice is as such:

- A linear component provided by potential theory (wave radiation);
- A quadratic component including a relative velocity formulation (buoy motions, flow velocity field), where the fluid velocity is assumed to be the particle velocity in the undisturbed incoming waves.
- However, an absolute velocity formulation could be used for a smaller buoy displacement (typically < 1500 tons) for which reduced diffraction / drag cancellation effect is observed.

When a relative velocity approach is chosen, a Morison damping formulation is applied to the CALM buoy as follows:

- Discretizing the buoy hull with horizontal slices and the absolute or relative velocity is applied at each slice level (Figure 1, top) to derive the surge load,
- Locating horizontal disks at the skirt level (Figure 1, bottom) to derive the vertical load,
- Using the buoy motions velocity + wave kinematics (so-called relative velocity assuming that there is no change in the flow due to diffraction and radiation of the incoming waves),
- Combining surge and heave loads to provide the pitch moment.

In the same way, the hydrodynamic force in mooring lines and risers is expressed by the Morison formula. Specifically, the drag force is proportional to the product of the relative velocity of the fluid with respect to the structure multiplied by its norm.

When an absolute velocity approach is conducted, the damping is defined by matrices with a linear and a quadratic term. In any case, it is necessary to introduce a linearization as indicated in Eq. (9).

$$\left\|v_{rel}\right\| v_{rel} \approx \Omega(v_{rel})\, v_{rel}\,. \tag{9}$$

The linearization coefficients are detailed in Kroliwkowski *et al.* (1980). If current is present, the coefficients are dependent on the current velocity. There is also an effect on the static force when waves and current are present. The results in the following sections are computed without current. In such a case, the coefficient is expressed as:

1. $\Omega = \dfrac{8}{3\pi} A$ in regular waves, with A the norm of the relative velocity,
2. $\Omega = \sqrt{\dfrac{2}{\pi}}\, 2\sigma$. in irregular wave, where σ is the Standard deviation of the relative velocity.

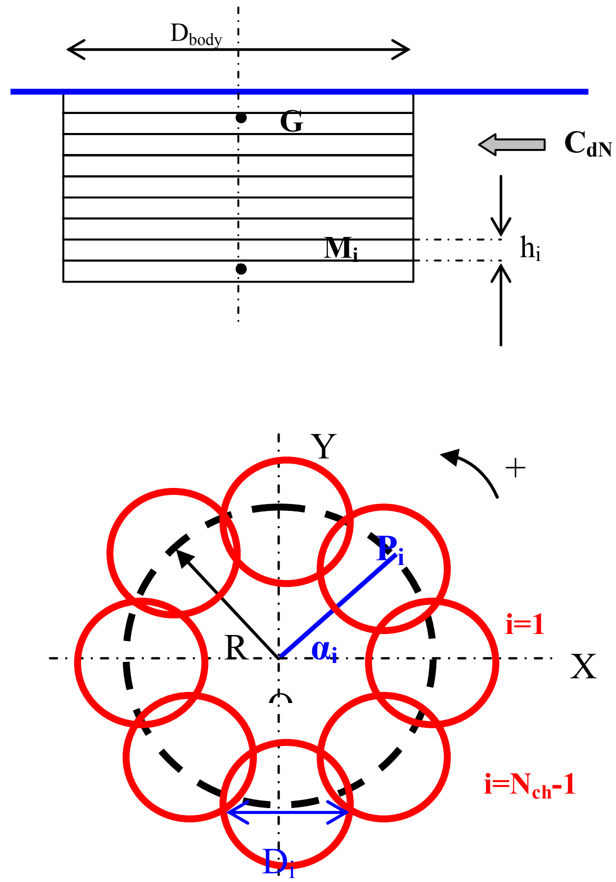


Figure 1: Hull buoy discretization used to derive surge, heave and pitch drag loads in FEA models.

The main difference between the regular and irregular wave calculations for the frequency domain approach is that the response at each frequency is decoupled for regular waves, while for irregular waves, σ links all the frequencies.

As the damping is a driven parameter for the buoy's motions, frequency domain analysis has to be validated with respect to time domain approach and model tests results.

COMPARISON BETWEEN TIME DOMAIN AND FREQUENCY DOMAIN ANALYSIS

In a first case, a complete CALM buoy offloading system is defined with U-shape export lines. The mooring system is a semi-taut system. Two oil offloading lines are also modeled. The lines connect the tanker to the buoy. The tanker is considered as an imposed boundary condition. A sketch of the model is presented in Figure 2.

For this model, three methods are tested to derive RAOs:

- Time domain (TD) calculations with regular waves,
- Time domain calculations with irregular waves performed over 2000s,
- Frequency domain (FD) calculations with regular waves.

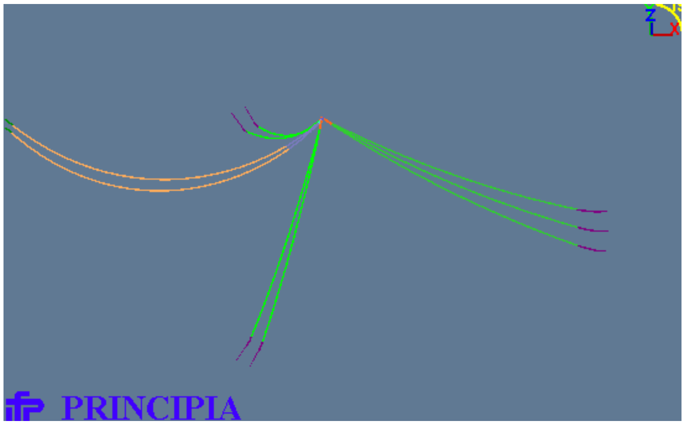


Figure 2: Sketch of the model: buoy, mooring lines and export lines.

A relative velocity formulation is first used with the same set of damping coefficients for all these analyses (time domain as well as frequency domain). Diffraction-radiation calculations are performed with Diodore (2005).

The reference curve is the RAO obtained through a time domain simulation performed over 3 hours (10,800s) with a JONSWAP spectrum (red curve "Reference" in Figure 3).

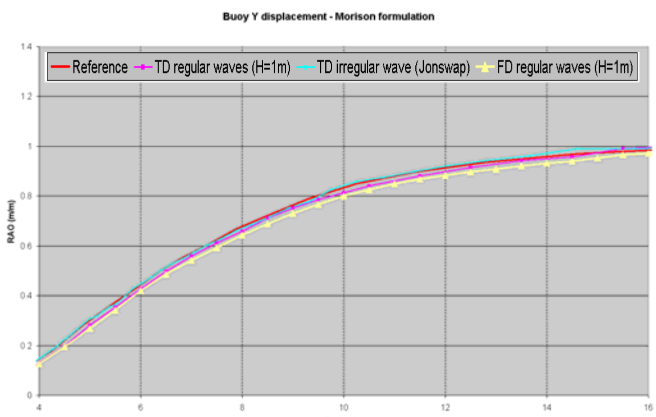


Figure 3a: Buoy sway RAOs with relative velocity approach (Morison damping formulation).

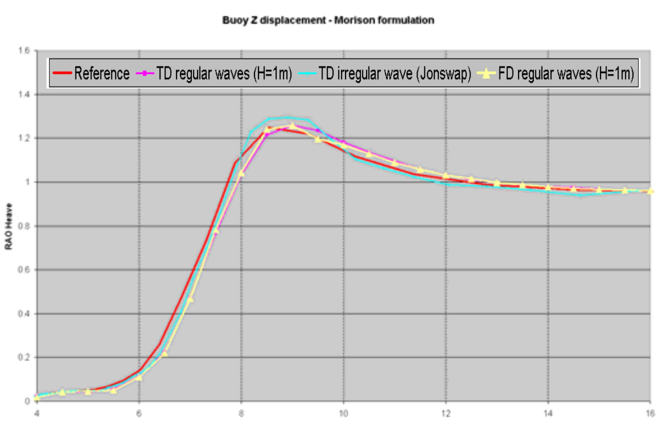


Figure 3b: Buoy heave RAOs with relative velocity approach (Morison damping formulation).

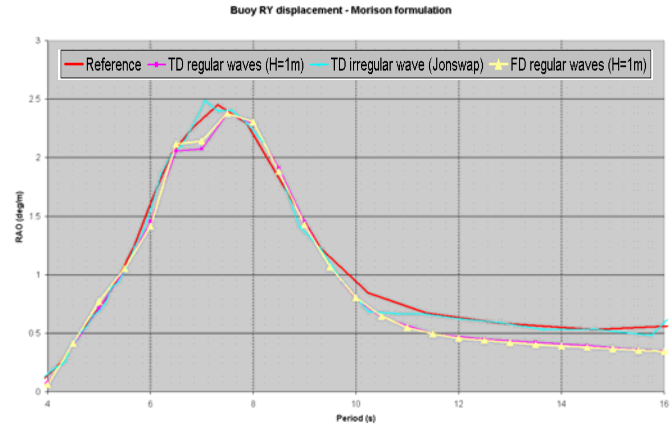


Figure 3c: Buoy pitch RAOs with relative velocity approach (Morison damping formulation).

On this real case, frequency domain analysis provides results that are consistent with the time-domain simulations. The motion RAOs can be used to perform parametric study and optimize the coupled system or as an input to time domain simulation by imposing the RAOs instead of performing a fully coupled analysis. As a final note, frequency analysis can also be used to provide an estimate of the fatigue in the export lines. As shown in Figure 4, prediction of the tension RAOs is also accurate.

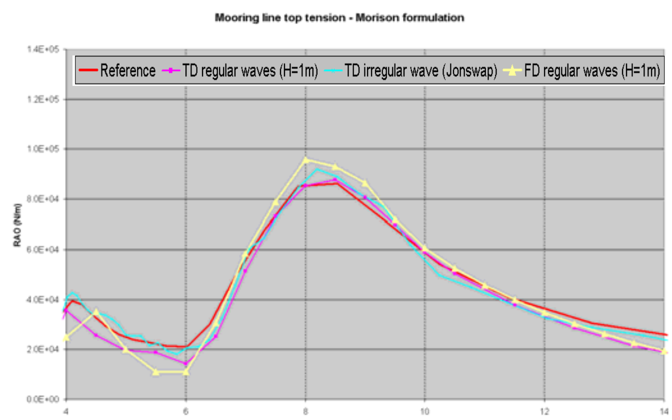
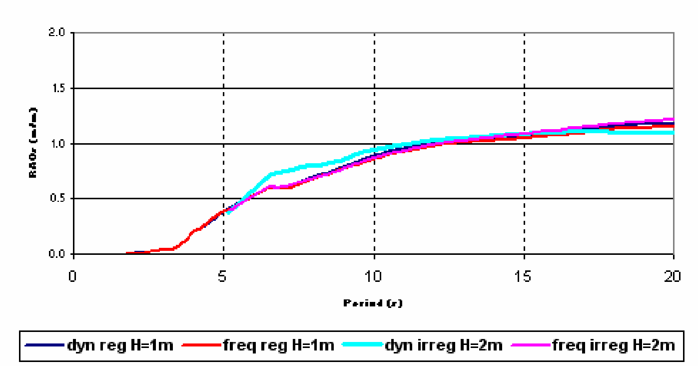


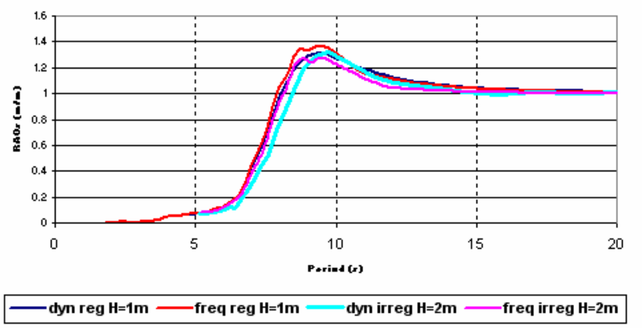
Figure 4 : Mooring line RAOs of tension with relative velocity approach (Morison damping formulation)

Now, the viscous damping on the buoy is defined thru a quadratic damping matrix, defined in the frame of reference of the buoy. The damping is computed with respect to the buoy velocity. At each period, a hydrodynamic database computed with Diodore (2005) provides the added mass and linear radiation damping. The following runs are performed:

1. Time domain simulation over 20 periods in regular waves (for each period),
2. Time domain simulation for an hour with a JONSWAP spectrum (Hs=2m, Tp=13s, at 0 degree with respect to the export lines),
3. Frequency domain calculations in regular waves,
4. Frequency domain calculations with the same JONSWAP spectrum.

Results for coupled buoy motion are reported in Figure 5. Again, very good agreement is found between time and frequency domain analysis. It is interesting to note that both the time domain and the frequency domain predict the same pitch cancellation.





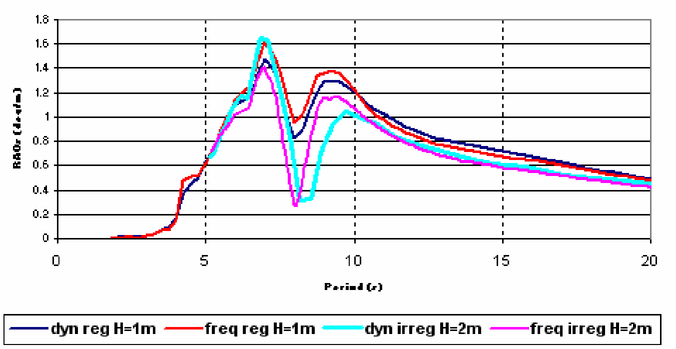


Figure 5 : RAOs in surge, heave, and pitch of the buoy.

COMPARISON WITH MODEL TEST

The model test program was specifically designed to study the deepwater buoy at a large scale. Due to the large typical prototype depth (greater than 1,000m) and the limitation of the basin facility the decision was made to model the buoy and the environment at a large model scale. The mooring system was represented by a simplified anchor leg system that resulted in similar stiffness and natural periods as a prototype buoy in 1,000 meters of water.

Two sets of model tests were performed: (1) a freely floating buoy (very soft springs used) and (2) a moored one. The freely floating tests were designed to provide data for the response of the buoy with no mooring influence to allow direct validation of the buoy hull model response. The second set of tests was performed with a simplified mooring system to provide data for the response of the buoy influenced by the mooring system. Model tests were conducted in the Offshore Engineering Basin at the Institute for Marine Dynamics in Canada.

Details and results for the experimental measurements can be found in Ryu et al. (2006). A short description of the main data is provided.

Experimental Set-up

The tank is 75m long by 32m wide with a variable water depth of up to 3m. The wavemakers consist of 168 rectangular panels across the front of the tank and along the side in an “L” formation. The buoy hull for the test is modeled at a scale of 1:35.6. For the moored tests the pretension of the mooring system resulted in a draft of 5.65m. The model is fitted with a skirt which has 18 holes (See Figure 6 and Table 1.).

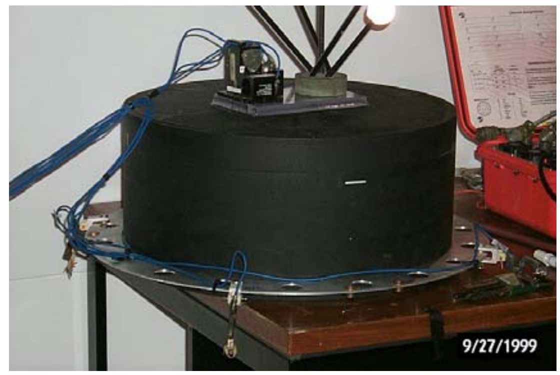


Figure 6: Deepwater buoy model.

Table 1: Buoy model particulars.

	Unit	Mooring 1	Mooring 2
Model Test Scale		35.6	35.6
Water Depth	m	106.8	106.8*
Buoy Hull Diameter	m	17.0	17.0
Skirt Diameter	m	21.0	21.0
Buoy Height	m	7.65	7.65
Draft	m	5.65	5.65
Weight in Air	ton	1293.2	878.6
KG	m	3.84	3.40
Buoy Total Rxx	m	3.82	4.39
Buoy Total Ryy	m	3.82	4.39
Fairlead Radius	m	9.50	9.50
No. of Mooring Legs		4	4

The first mooring configuration was designed to investigate the motions of a freely floating buoy with minimal influence of the mooring system. The horizontal mooring system consisted of a 4 lines with soft springs that maintained the buoy at the desired location but had minimal feedback to the wave frequency motions. The second mooring system configuration was designed to have the stiffness characteristics and pretension of a 1,000m mooring system for an offloading buoy. To simplify the mooring (and its modeling) the mooring was represented by four legs spaced 90 degrees apart, with fairlead angles of 45 degrees (see Figure 7). The pretension was designed to provide the net mooring and offloading system load on the buoy (to give the desired draft of 5.65m). The mooring system was further simplified to be as light as possible with minimal drag so that the stiffness was primary influence on the buoy response. Details of the two systems are provided in Table 2.

Table 2: Particulars of the two mooring configurations.

	Unit	Mooring 1	Mooring 2
Length	m	350	133.3
Wet Weight	kg/m	NA	3
Diameter	mm	NA	NS
EA	MT	180	1963
Pretension	MT	22	150
Fairlead Angle	deg	0	45

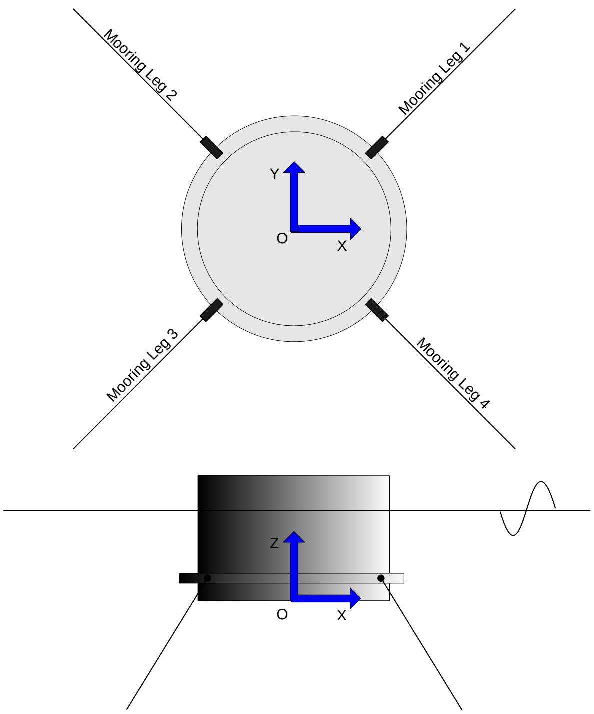


Figure 7: Sketch of the coupled system.

Free floating configuration

A series of comparisons are made between the numerical simulations and the model test results. The motion RAOs of the buoy were derived at the buoy CoG. The waves are incident head-on for all cases. Data obtained from the model tests and shown in the figures is extracted from six different seastates with peak periods, Tp = 4.5, 6, 8, 9, 11 and 15 seconds, respectively.

Previous comparisons (Ryu *et al.*) were conducted with a diffraction analysis code with and without modeling the skirt viscous damping. To validate the new approach, a comparison is performed on the best modeling (i.e. with skirt damping modeling). In Figure 8 to Figure 10, the surge, heave, and pitch of the buoy are computed. As a reference the previous results are given with and without skirt damping, together with the new results with skirt damping. Calculations are performed with two sets of regular waves with respectively a 1m and a 0.5m wave height. The present frequency approach gives results very similar to the previous method.

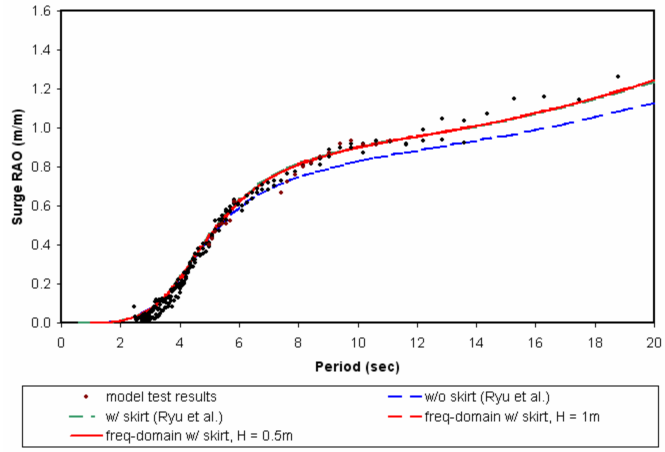


Figure 8 : Surge RAOs for free-floating case.

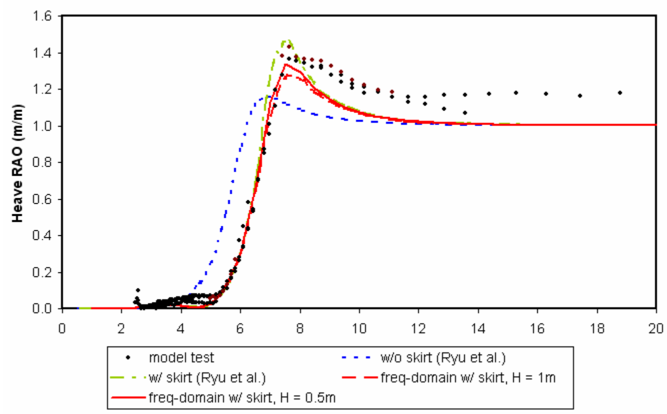


Figure 9 : Heave RAOs for free-floating case.

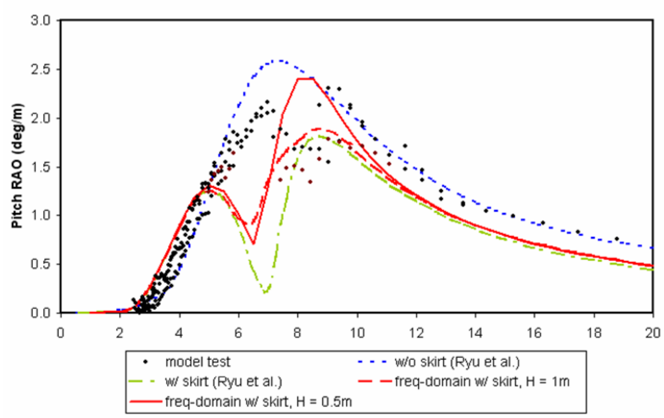


Figure 10 : Pitch RAOs for free-floating case.

Finally a comparison is provided in between frequency domain and time-domain the pitch using 1m regular waves. Frequency domain and time domain calculations are in very good agreement.

Moored configuration

Comparisons are now conducted on the moored buoy configurations. Viscous damping is imposed following the JIP guidelines without further benchmarking of the coefficients.

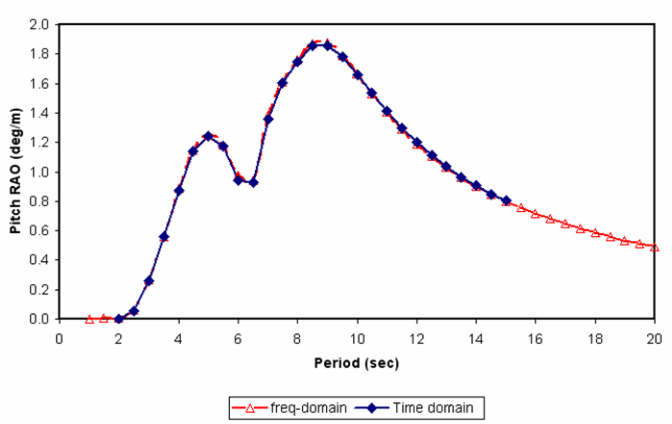


Figure 11 : Comparison between time domain and frequency domain analysis for the pitch motion.

Calculations are performed on the fully coupled system. Regular waves are used with one and two meters wave height. Irregular waves simulations are performed with three different peak periods: 4.5s, 8s, and 15s. Time domain calculations with regular waves are also provided for reference. The RAOs in surge, heave and pitch are then computed and compared to measurements, respectively in Figure 12, Figure 13, and Figure 14. Good agreement is obtained between calculations and experimental data for all directions of motion.

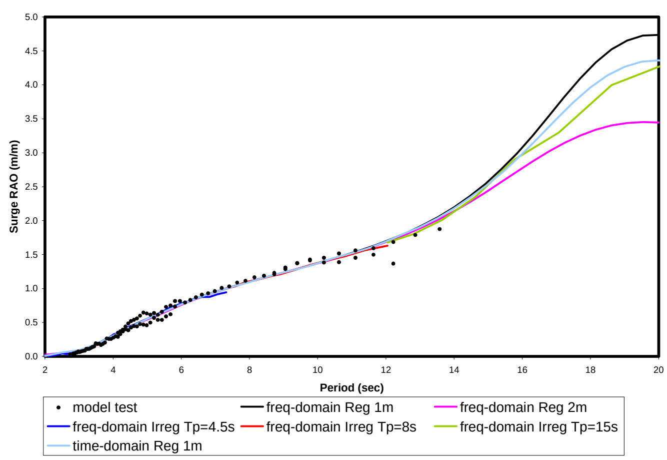


Figure 12: Surge RAOs for moored case.

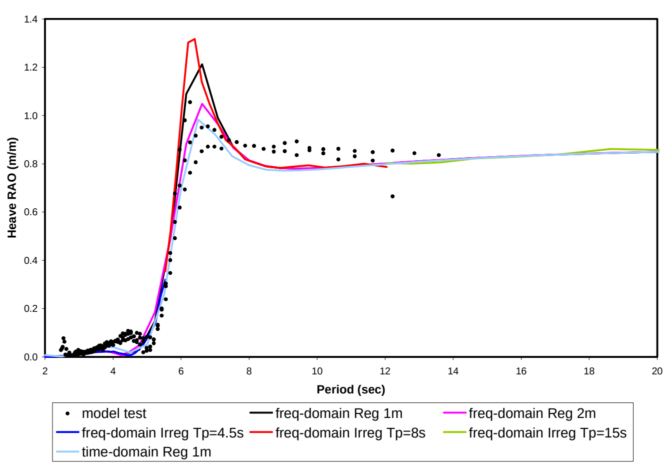


Figure 13: Heave RAOs for moored case.

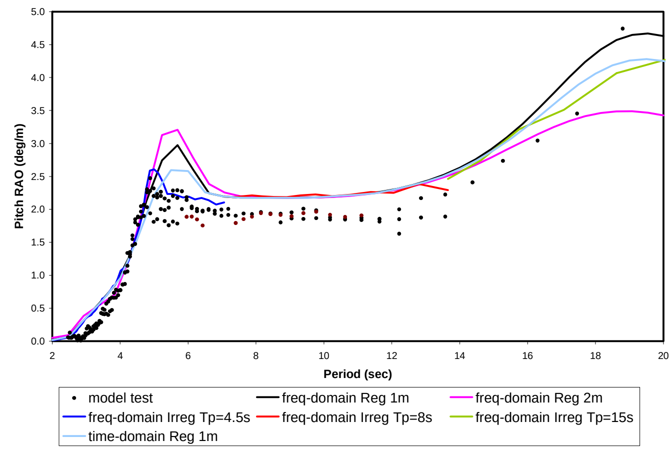


Figure 14: Pitch RAOs for moored case.

CONCLUSION

A frequency domain analysis has been applied to the derivation of RAOs for CALM buoys with mooring and export lines. Comparison with time domain simulations shows that the results are very reliable at a fraction of the computer time. Such an approach, restricted to first order wave load is specifically useful for parametric study and can help designing the coupled system. Motions but also tension and bending RAOs in the mooring and export lines are also well captured. Therefore, it is possible to use the frequency domain analysis to quickly estimate the performance of the system in terms of fatigue life.

Comparison with experimental results shows good agreement between the frequency domain calculations and the measurements. As viscous damping is one of the key elements to model buoy coupled motion, it indicates that both the damping methodology and its linearization are properly taken into account.

REFERENCES

Colbourne, DB (2000). "Deep Water CALM Buoy Moorings Wave Basin Model Tests," *Technical Report*, Institute for Marine Dynamics, National Research Council Canada, TR-2000-06.

DeepLines Theory Manual, V4.1, 2004.

Diodore V3r2 theory manual, 2005

Duggal, A.S. and Ryu, S. (2005), "The dynamics of deepwater offloading buoys," *Fluid Structure Interactions and Moving Boundary Problems*, WIT Press 2005, pp. 269-278.

Duggal, A.S., Ryu, S., Heyl, C.N., and Lee, D.H. (2006), "Mooring buoys in the offshore oil/gas industry and their global motion analysis," *2006 U .S.-Korea Conference*, Teaneck, New Jersey.

Heurtier, J.-M., "Analyse dynamique fréquentielle," report for project CEP&M M2113/97.

JIP CALM Buoy Phase II Technical Report (2005). CTR1-Buoy hydrodynamics – Model tests analysis, Report No. Ret5.5.005.01.

Kroliwkowski, L.P. and Gay, T.A., "An improved linearization technique for frequency domain riser analysis," Proceedings of OTC 1980.

Ricbourg, C. (2005), "Mise en place d'une résolution fréquentielle dans DeepLines," Internal Note, Principia R.D.

Ricbourg, C., Berhault, C., Camhi, A., Lecuyer, B., Marcer, R., "Numerical and Experimental Investigations on Deepwater CALM Buoys Hydrodynamics Loads, Proceeding of OTC 2006.

Ryu, S., Duggal, A.S., Heyl, C.N., Liu, Y., "Prediction of Deepwater

Oil Offloading Buoy Respeonse and Experimental Validation," *Int. J. of Offshore and Polar Engineering*, pp 290-296, Dec. 2006.

Valliet, F, Cuenca, A, and Le Buhan, P (2002). "JIP CALM Buoy Model Test Results and DeepLines Calibration," *Technical Report*, Principia R.D. Ret.25.084.01.

Huang, Z.J., Santala, M.J., Wang, H., Yung, T.W., Kan, W. and Sandstrom, R.E. (2005) "Component Approach for Confident Predictions of Deepwater CALM Buoy Coupled Motions – Part I: Philosophy". OMAE2005-67138. Proc. of 24th OMAE Conference, June 12-17, 2005, Halkidiki, Greece.

Santala, M.J., Huang, Z.J., Wang, H., Yung, T.W., Kan, W. and Sandstrom, R.E. (2005) "Component Approach for Confident Predictions of Deepwater CALM Buoy Coupled Motions – Part II: Analytical Implementation". OMAE2005-67138. Proc. of 24th OMAE Conference, June 12-17, 2005, Halkidiki, Greece.