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Experimental Investigation of Wave Drift Damping and Slow Drift Motion in Bi-Frequency Domain

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INTRODUCTION

The response of large moored vessels and floating offshore structures to irregular seas is often dominated by large-amplitude, low-frequency quadratic nonlinear motions. As the use of such soft-moored systems for deep-water drilling, production and storage facilities is rapidly increasing, the development of more accurate numerical prediction tools, of which experimental verification is an integral part, becomes ever important.

Dalzell (1976) conducted the first model tests to determine the quadratic frequency response function (QFRF) for added wave resistance in random seas. QFRF for added resistance was theoretically analyzed by Dalzell and Kim (1976) and applied to the prediction of surge drift motion of a moored ship in random head seas by Kim and Breslin (1976). Slow drift motion of a moored vessel in random waves has usually been predicted in the time domain by solving an equation of drift motion using random drift force and damping coefficients estimated from experimental studies employing mono-chromatic waves—for instance, by Wichers (1987).

The objective of the present research is to develop an improved technique for prediction of slow drift motion due to random sea excitation by utilizing complete drift force and motion QFRFs and bi-frequency domain wave drift damping coefficients, as dictated by Volterra input-output theory. Given that second-order response in random seas is sensitive to the nonlinear interaction of many combinations of wave frequencies, it was considered important first to conduct a deterministic experimental investigation of the QFRF for slow drift motion of an idealized vessel/mooring system in a series of bi-chromatic waves. Further details of the experimental and theoretical investigation are given by Krafft and Kim (1990).

VOLTERRA MODEL

The quadratic Volterra polynomial is a model by which the slow drift force or motion of a large floating body subject to stationary Gaussian seas can be analyzed.

Utilizing the assumed symmetries of the QFRF in the bi-frequency plane, one needs only to consider the octants on either side of the positive ω_1-axis as shown in Fig. 1. In these octants, ω_1 is positive and $\omega_1 > |\omega_2|$. Wave frequency coordinates (ω_1, ω_2) are mapped into difference and sum frequency coordinates (Ω_1, Ω_2) as follows:

$$\Omega_1 = \omega_1 - \omega_2, \qquad \Omega_2 = \omega_1 + \omega_2 \tag{1}$$

Given simple bi-chromatic (dual) wave excitation defined by:

$$\eta(t) = a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t), \qquad \omega_1, \omega_2 > 0 \tag{2}$$

the quadratic Volterra model yields an expression for slow drift vessel response comprised of mean and low frequency terms:

$$D(t) = \frac{1}{2}\left\{a_1^2 \cdot G_2(\omega_1, -\omega_1) + a_2^2 \cdot G_2(\omega_2, -\omega_2)\right\} + \mathrm{Re}\left\{a_1 a_2 \cdot G_2(\omega_1, -\omega_2) \cdot e^{i(\omega_1 - \omega_2)t}\right\} \tag{3}$$

where a_1, a_2 are dual wave component amplitudes and G_2 is the QFRF for drift force or motion.

Consider the slow surge drift motion of a soft, linearly moored vessel excited by low-frequency second-order forces resulting from dual wave and vessel interaction. For this type of vessel/mooring system, the low-frequency response will be represented in the lower octant of the bi-frequency domain, where ω_2 is always negative and ω_1 is always positive. Hence, $\Omega_1 = \omega_1 + |\omega_2|$ and $\Omega_2 = \omega_1 - |\omega_2|$ and will hereafter be referred to as sum and difference frequencies, respectively.

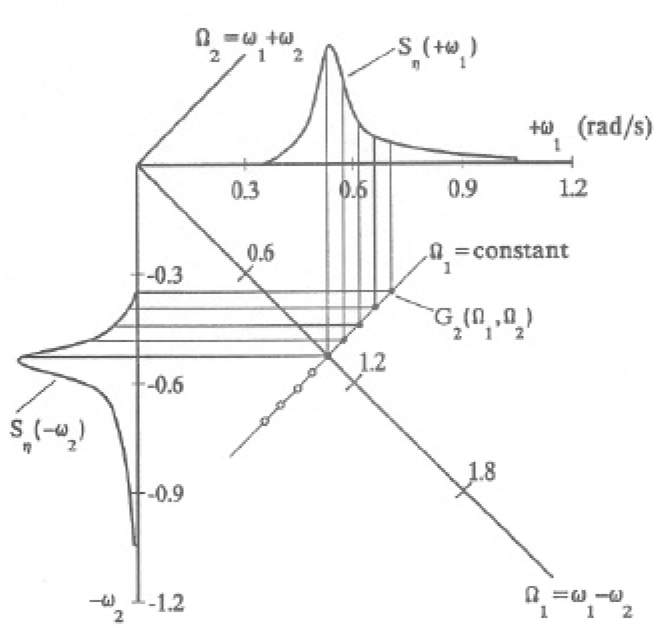

Fig. 1 Mapping of bi-frequency plane

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Wave drift force is the excitation to the moored system that produces difference frequency quadratic surge response in the same lower octant of the bi-frequency domain. Damping forces are assumed to vary linearly with low-frequency surging velocity, and mooring system forces are assumed to vary linearly with surge displacement. One may define wave drift force as the second-order force acting on a body that is restrained from drift motion, but allowed to undergo linear motions. The drift force, R, of a vessel drifting with speed, U, is expressed as a function of bi-frequency and speed, i.e., $R(\Omega_1, \Omega_2, U)$. Exploiting the smallness of low-frequency surging velocity, R is expanded in a Taylor series about zero speed:

$$R(\Omega_1,\Omega_2,U) \simeq R(\Omega_1,\Omega_2,0)+U\cdot\frac{\partial R}{\partial U}(\Omega_1,\Omega_2,0) \quad (4)$$

The first term in Eq. 4 is the ordinary drift force. The second term is regarded as the wave drift damping force in which $\partial R(\Omega_1, \Omega_2, 0)/\partial U$ is the wave drift damping coefficient. Applying the foregoing assumptions, an equation for the drift motion, x, due to drift force, X, may be written:

$$(M+M_a)\cdot\ddot{x}+N_t\cdot\dot{x}+K\cdot x=X \quad (5)$$

where

$$x=x_o\cdot e^{-i\Omega_2 t}, \qquad x_o=a_1a_2\cdot G_{2x}(\Omega_1,\Omega_2) \quad (6)$$

and

$$X=X_o\cdot e^{-i\Omega_2 t}, \qquad X_o=a_1a_2\cdot G_{2X}(\Omega_1,\Omega_2,0) \quad (7)$$

In Eq. 5, M, M_a, N_t and K are the vessel mass, added mass, total damping and mooring stiffness, respectively. Use of Eqs. 6 and 7 in Eq. 5 yields the following relation between the QFRF for drift motion, G_{2x}, and the QFRF for drift force, G_{2X}:

$$G_{2x}(\Omega_1,\Omega_2)=H(\Omega_2)\cdot G_{2X}(\Omega_1,\Omega_2,0) \quad (8a)$$

where

$$H(\Omega_2)=\left[-\Omega_2^2(M+M_a)-i\Omega_2N_t+K\right]^{-1} \quad (8b)$$

is a linear frequency response function depending only on Ω_2 for a section of the bi-frequency domain defined by constant Ω_1. The QFRFs for drift force and motion are nondimensionalized using vessel beam, B, and displacement, Δ, as follows

$$\overline{G}_{2X}=\frac{G_{2X}\cdot B^2}{\Delta}, \qquad \overline{G}_{2x}=G_{2x}\cdot B \quad (9)$$

Replacing R in Eq. 4, with X_o from Eq. 7, one obtains a bi-frequency dependent wave drift damping coefficient:

$$N_w(\Omega_1,\Omega_2,0)=a_1a_2\cdot\frac{\partial G_{2X}(\Omega_1,\Omega_2,0)}{\partial U} \quad (10)$$

Total damping, N_t, is then the sum of a wave drift component, N_w, and a viscous component, N_v.

EXPERIMENTAL INVESTIGATION

Experimental measurements were conducted in the two-dimensional wave tank at Texas A&M University, which is 37 m long, 0.91 m wide and 1.22 m deep and is equipped with a hinged-flap wave maker. Model tests were performed in a simulated water depth of 91.0 m using a 1:100 Froude scale shallow-drafted rectangular barge configured with a linear spring mooring system and rotary potentiometer for measurement of surge motion. Particulars of the barge are given in prototype scale in Table 1.

Length	L	m	71.12
Beam	B	m	35.56
Depth	D	m	12.70
Draft	T	m	4.16
Displacement	Δ	tf	10,506.
Mass	M	tf · s²/m	1,071.
Added mass (zero frequency)	M_a	tf · s²/m	243.
Center of gravity above keel	KG	m	6.55
Longitudinal metacentric height	GM	m	96.96
Longitudinal radius of gyration (in air)		m	26.0
Total mooring lines restoring coefficient	K	tf/m	2.42
Viscous damping coeff.	N_v	tf · s/m	4.57
Surge natural period	T_x	s	143.4
Model scale	λ		(1:100)

Table 1 Particulars of barge and mooring system

Damping QFRF from Regular and Dual Wave Tests

For a lightly damped, linear spring-mass system, the viscous damping coefficient may be expressed as:

$$N_v \simeq \frac{\delta_v K}{\pi\omega_x} \quad (11)$$

From calm water surge oscillation decay tests, the logarithmic decrement and the natural frequency of surge oscillation were found to be $\delta_v = 0.26$ and $\omega_x = 0.0438$ rad/s, respectively. Using Eq. 11, the resulting viscous damping coefficient is $N_v = 4.57$ tf · s/m.

Free surge oscillation decay tests were performed in monochromatic waves with frequencies in the range $\omega = 0.45$–0.90 rad/s ($\Omega_1 = 0.9$–1.8) for each of several small amplitudes. As the damping due to waves was found to vary linearly with wave amplitude squared, the QFRFs for mono-chromatic wave drift damping, $N_w(\Omega_1, 0, 0)/a^2$, are given by the resulting slopes for each frequency tested and are shown in Fig. 2.

Similar decay tests were performed in dual waves with varying component amplitudes. Sum frequencies were varied in the range $\Omega_1 = 0.9$–1.8 rad/s, while the difference frequency was held fixed

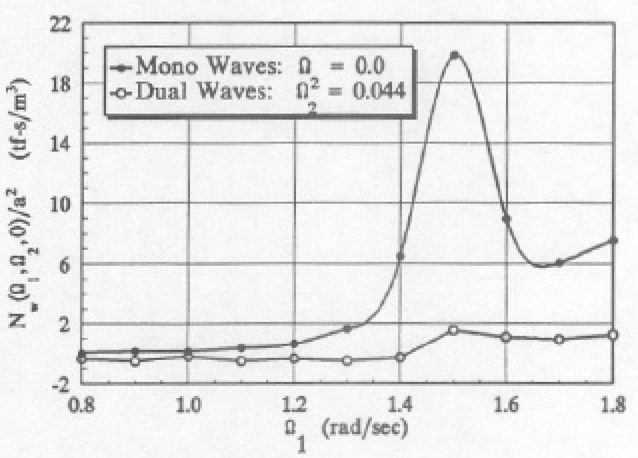

Fig. 2 Wave drift damping in monochromatic waves and in dual waves at surge resonance

at the natural frequency of surge oscillation, $\Omega_{2x} = 0.0438$ rad/s. Again, wave drift damping was found to vary approximately linearly with wave amplitude squared. The resulting damping QFRFs, $N_w(\Omega_1, \Omega_{2x}, 0)/a^2$, are shown in Fig. 2. Most notably, these values are often much lower than the mono-chromatic wave damping terms at equivalent frequencies. Although some dual wave damping coefficients have negative values, the total damping always remains positive.

Motion QFRF from Regular and Dual Wave Tests

In accordance with the Volterra model, the mean ($\omega_1 = -\omega_2$) QFRFs for surge motion are determined as the ratio of the steady surge offset in mono-chromatic waves to wave amplitude squared. As shown in Fig. 3, there is fair agreement between the measured and predicted mean QFRFs except at the sum frequency $\Omega_1 = 1.6$ rad/s, which is closest to a calculated critical transverse tank resonance frequency of $\omega_c = 0.82$ rad/s ($\Omega_1 = 1.64$ rad/s).

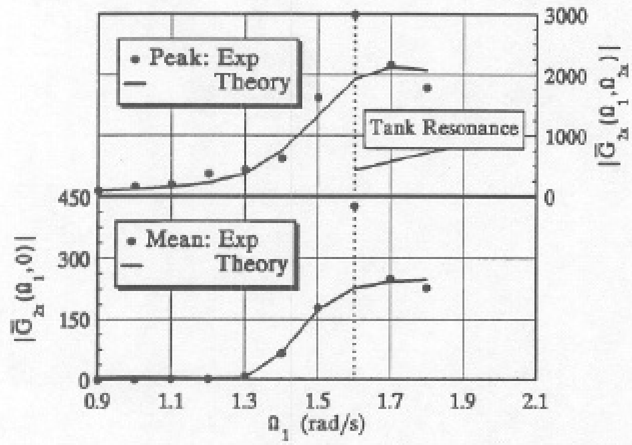

Fig. 3 Nondimensional moduli of mean ($\Omega_2 = 0$) and peak ($\Omega_2 = \Omega_{2x}$) QFRFs for surge drift motion

A series of 120 steady-state dual wave tests were conducted, covering a bi-frequency range of $\Omega_1 = 0.9$–1.8 rad/s and $\Omega_2 = 0.0$–0.125 rad/s. Reliable measurements at sum frequencies above $\Omega_1 = 1.8$ rad/s (18 rad/s model scale) become increasingly difficult to obtain for the present model and tank facilities. Particular frequency combinations were selected to give higher resolution in the region close to the surge natural frequency. Target component wave amplitudes were held equal and fixed for all tests corresponding to a given sum frequency.

The modulus and phase of the surge motion QFRF for two cross-sections of the bi-frequency domain are shown in Figs. 4 and 5. Also shown are two predictions obtained from the solution of Eqs. 8a and 8b, using analytically estimated QFRFs for drift force, and measured bi-chromatic or mono-chromatic wave drift damping coefficients. Fig. 4 corresponds to a cross-section for a relatively low constant sum frequency of $\Omega_1 = 0.9$ rad/s, where predictions using either bi-chromatic or mono-chromatic wave drift damping agree well with measurements. Fig. 5 depicts a section of the QFRFs for a constant sum frequency of $\Omega_1 = 1.4$, where the prediction using the mono-chromatic damping coefficient shows poor agreement with measurements. Throughout the remainder of the bi-frequency domain investigated, it was generally observed that predictions using mono-chromatic wave drift damping tended to severely underestimate the experiment when these coefficients were large compared to their bi-chromatic counter-

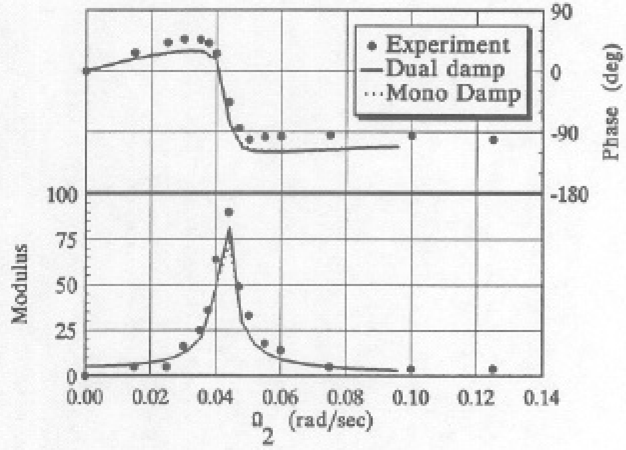

Fig. 4 Section of nondimensional QFRF for surge drift motion of barge at $\Omega_1 = 0.9$ rad/s

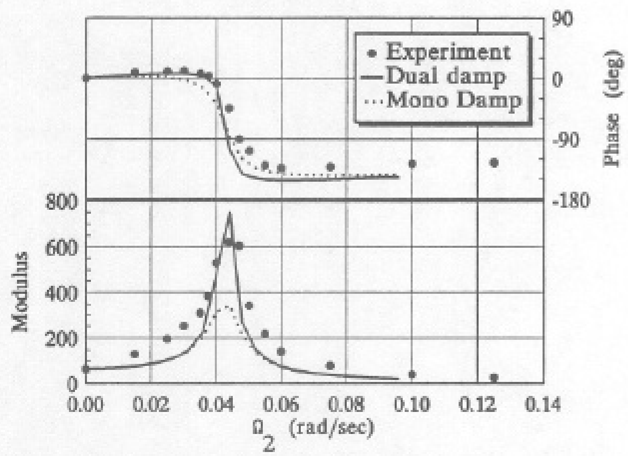

Fig. 5 Section of nondimensional QFRF for surge drift motion of barge at $\Omega_1 = 0.9$ rad/s

parts. These comparisons clearly illustrate the sensitivity of the present prediction method to damping.

The peak moduli of the measured QFRFs (Ω_2 = constant = 0.0438 rad/s) are shown together with predictions employing bi-chromatic wave damping in Fig. 3. Except near the transverse tank resonance frequency of $\Omega_1 = 1.64$ rad/s, there appears to be reasonable agreement. The damping coefficient used for the predictions at $\Omega_1 = 1.6$ was taken as a weighted average of the values at adjacent frequencies, as measurements involving frequencies near transverse tank resonance proved to be the least reliable.

The moduli of the QFRFs resulting from the dual wave tests are presented in bi-spectral form in Fig. 6. Measurements corresponding to $\Omega_1 = 1.6$ have been omitted because of their possible inaccuracy due to tank wall effects. The corresponding bi-spectral view of theoretical predictions based on bi-chromatic damping is given in Fig. 7.

SUMMARY AND CONCLUSIONS

A series of mono- and bi-chromatic wave drift damping coefficients were experimentally determined and employed in the solution of a second-order equation of surge drift motion in dual waves. Based on measurements conducted with size-limited experimental facilities, bi-frequency wave drift damping was found

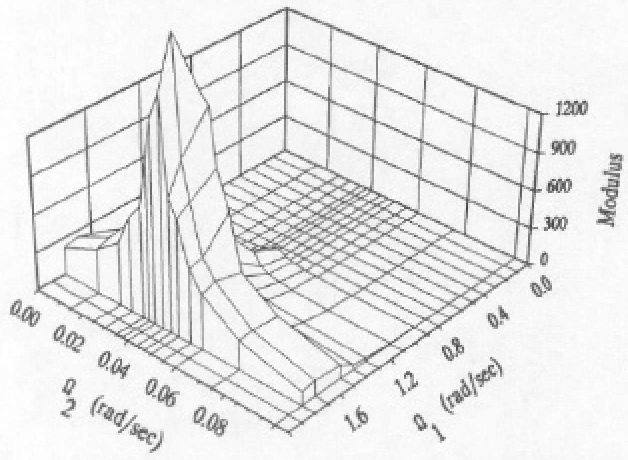

Fig. 6 Nondimensional modulus of experimental QFRF for barge surge drift motion

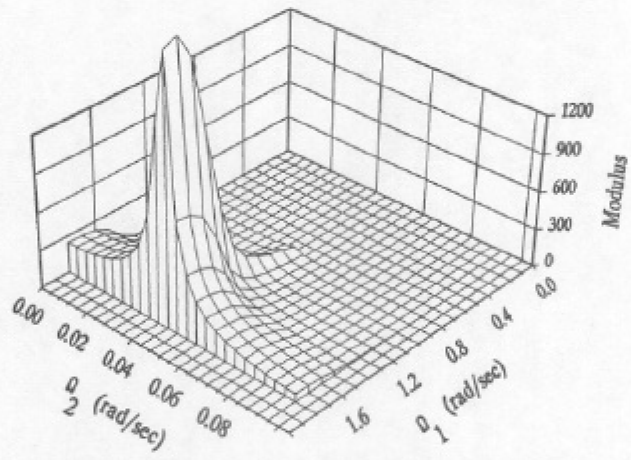

Fig. 7 Nondimensional modulus of theoretical QFRF for barge surge drift motion

to be approximately linearly proportional to low-frequency surging velocity. The experimental data also indicated an approximately linear proportionality to the product of the dual wave component amplitudes. The magnitude of the bi-chromatic wave drift damping coefficients obtained near surge resonance was often found to be considerably lower than mono-chromatic wave damping terms measured at equivalent sum frequencies. Utilization of bi-frequency domain wave drift damping coefficients for the prediction of QFRFs generally yielded acceptable agreement with experimental measurements.

It should be noted that the small size of the model causes difficulties in the scaling of viscous forces, and that friction in the mechanical measurement apparatus can produce errors in the measurement of low-frequency motions. Therefore, it is recommended that the foregoing results be verified in a larger experimental facility.

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