LINEARIZATION OF QUADRATIC DRAG TO ESTIMATE CALM BUOY PITCH MOTION IN FREQUENCY-DOMAIN AND EXPERIMENTAL VALIDATION

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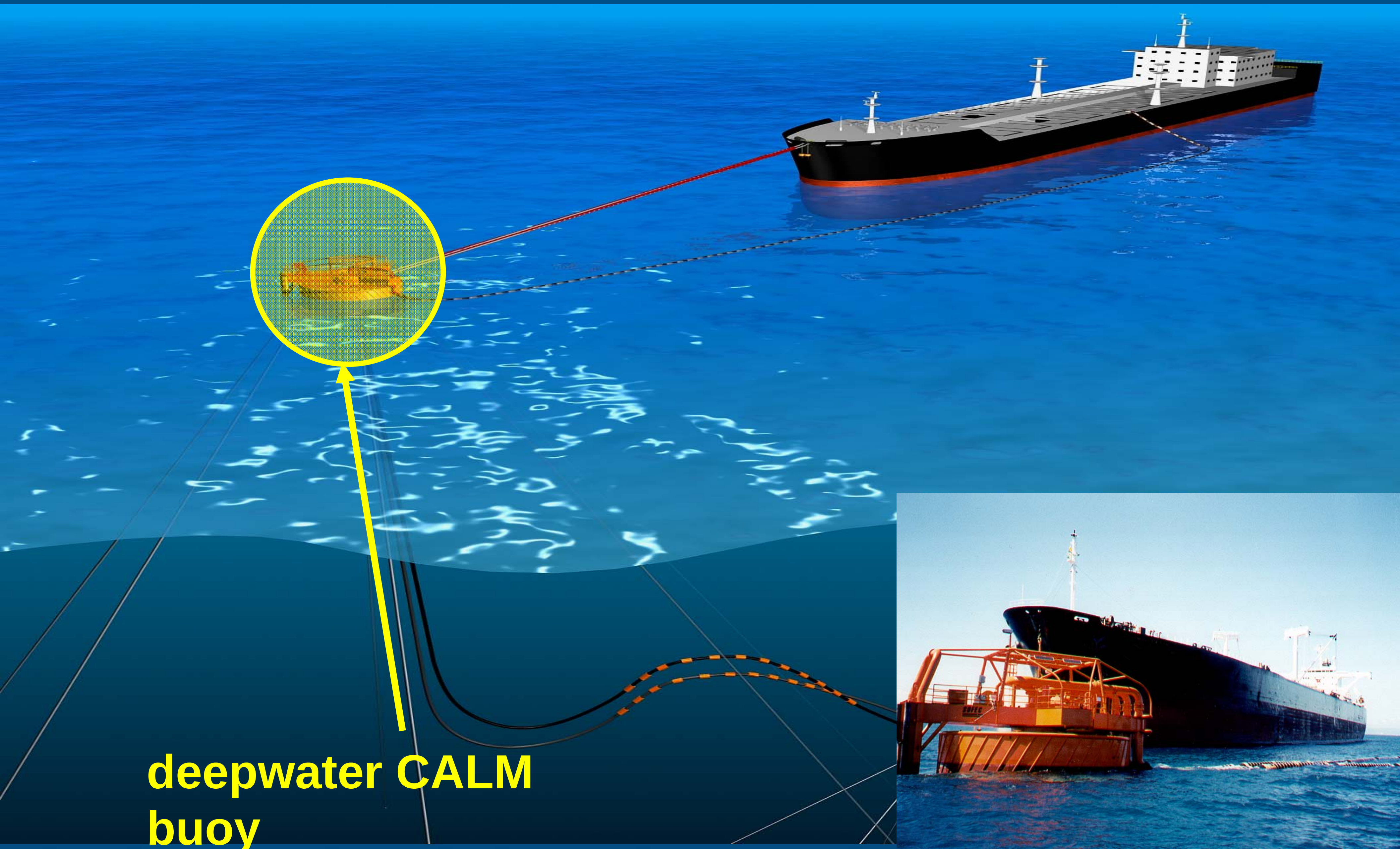
Outline

- Background
- Current Study (Pitch Motion Linearization)
- Discussion of Results
- Summary

Catenary Anchor Leg Mooring (CALM) Buoy

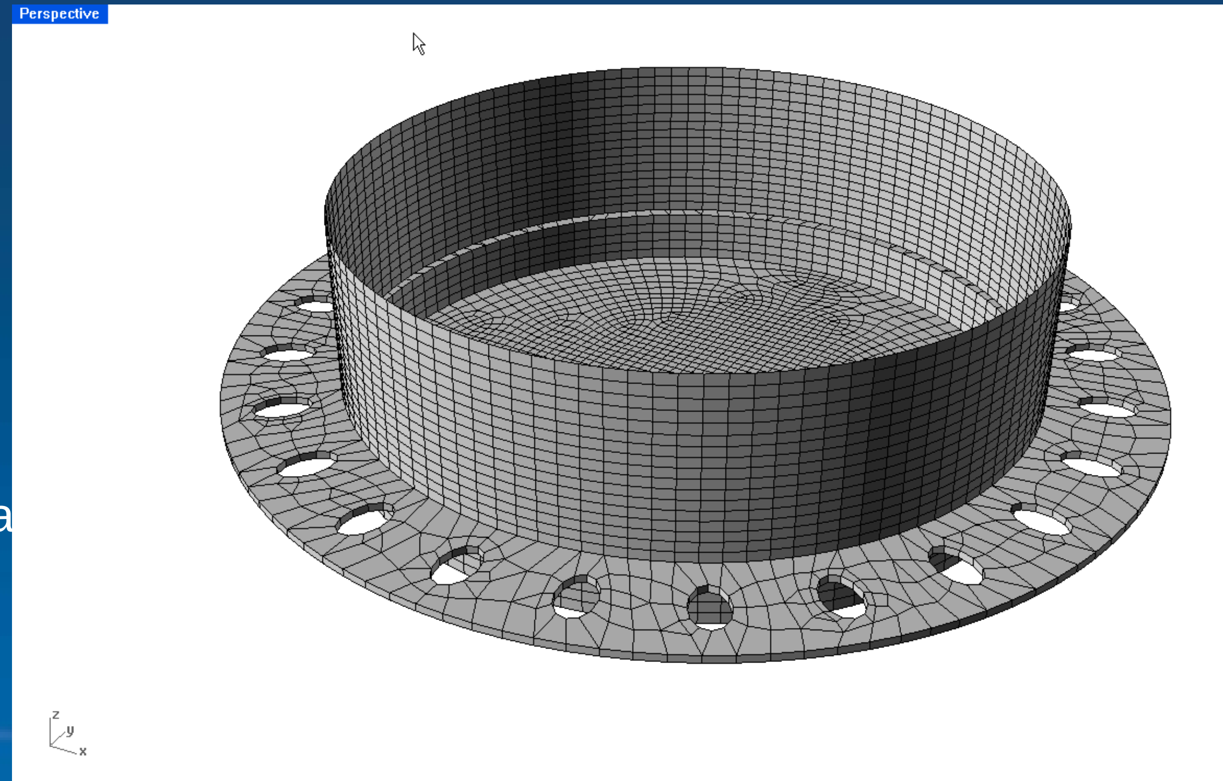


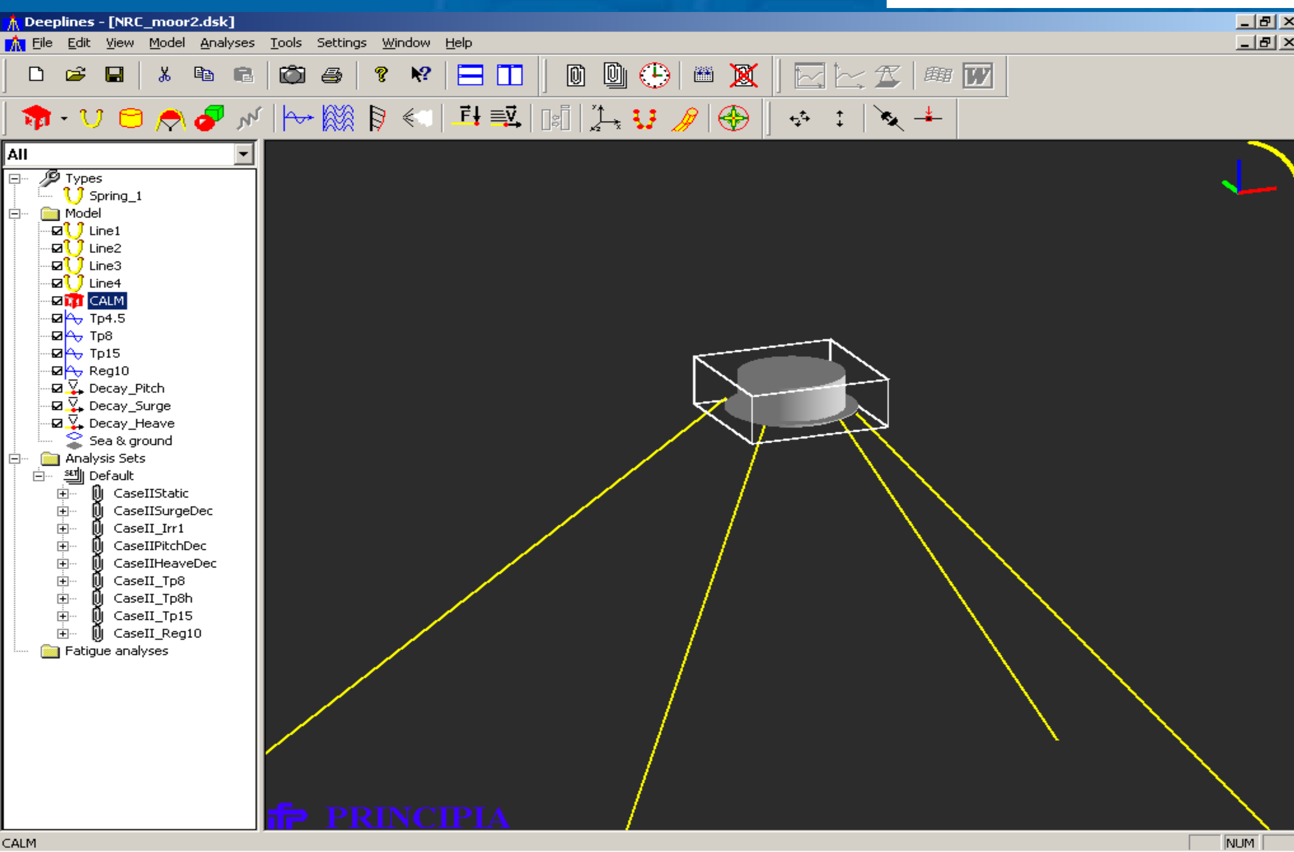
Methods for Wave Frequency Motion Estimate

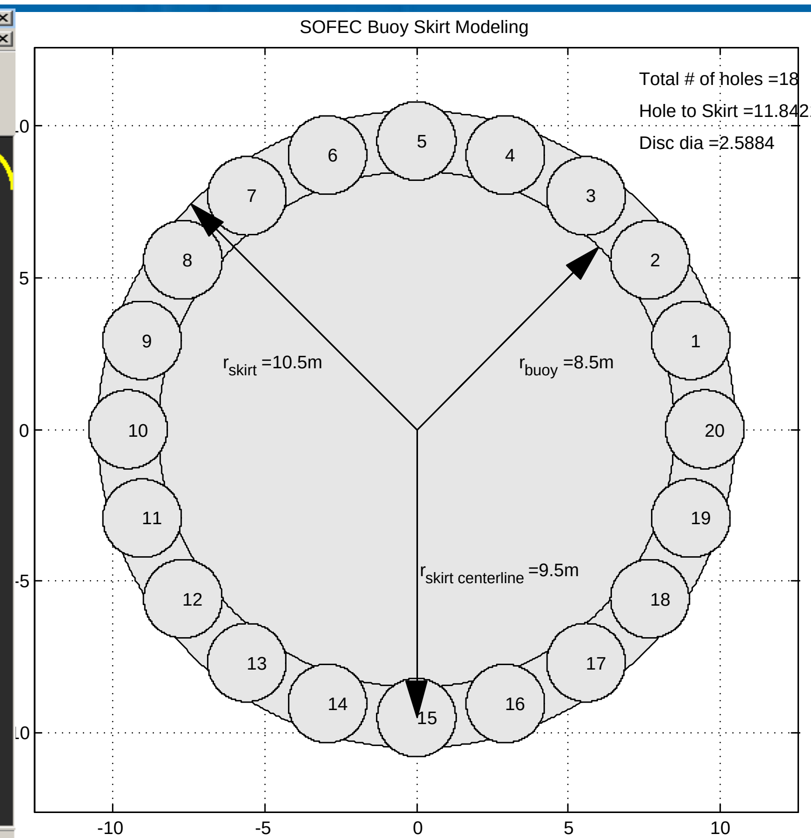
- Model Test
- Time Domain (TD)
- Frequency Domain (FD)
  - Quadratic Drag Linearization
    - Iterative
    - Stochastic

Time Domain Model

- Morison type disc elements
- Nonlinear drag force proportiona relative velocity squared





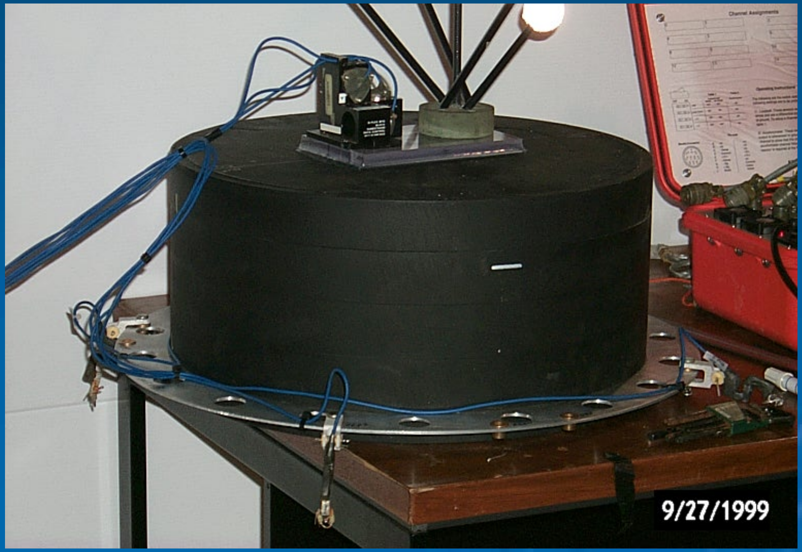


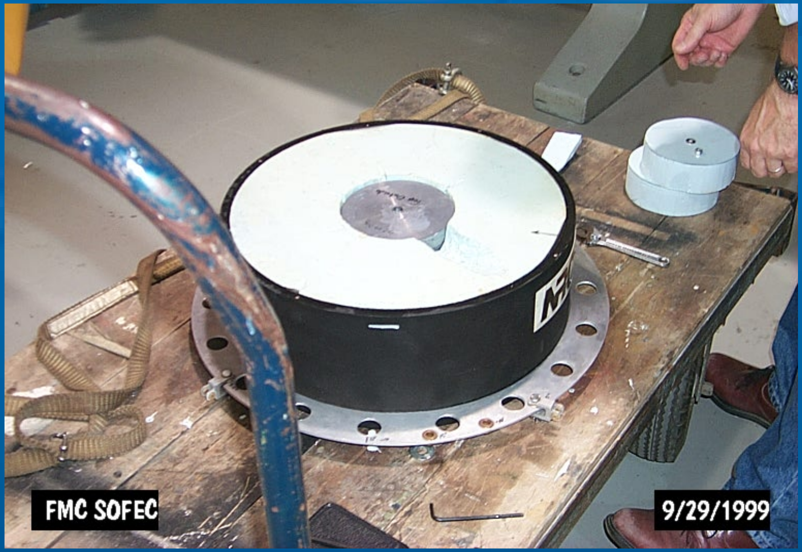
Model Test

- Model Particulars

- Model scale = 1:35.6
- Water depth = 106.8m
- Buoy dia = 17m
- Skirt dia = 21m
- Draft = 5.65m

Buoy Model (cont'd)





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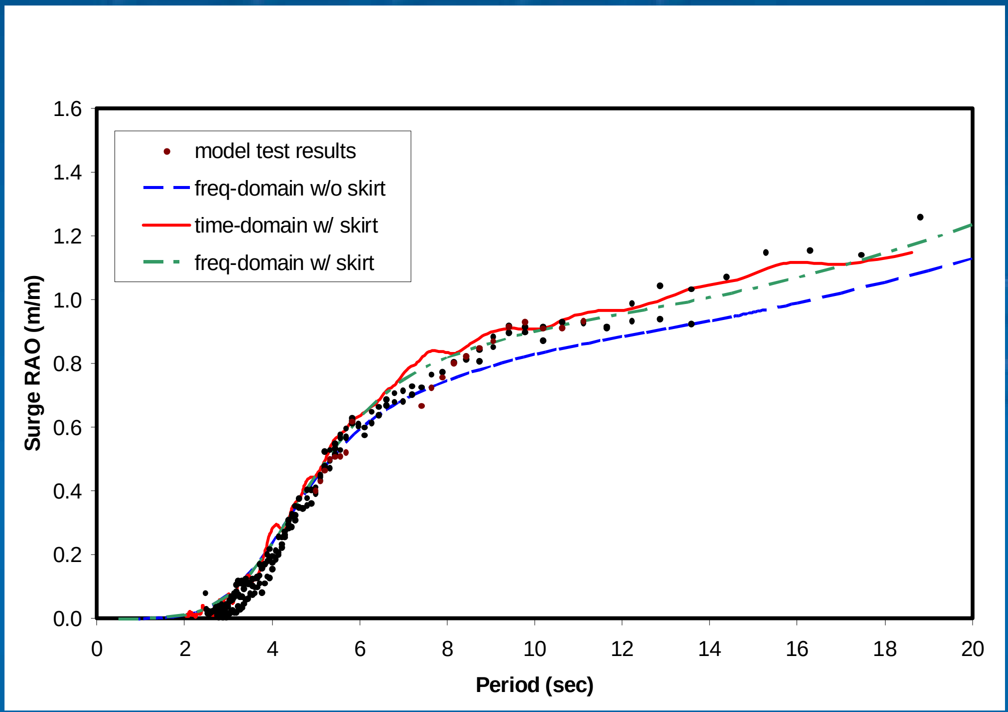
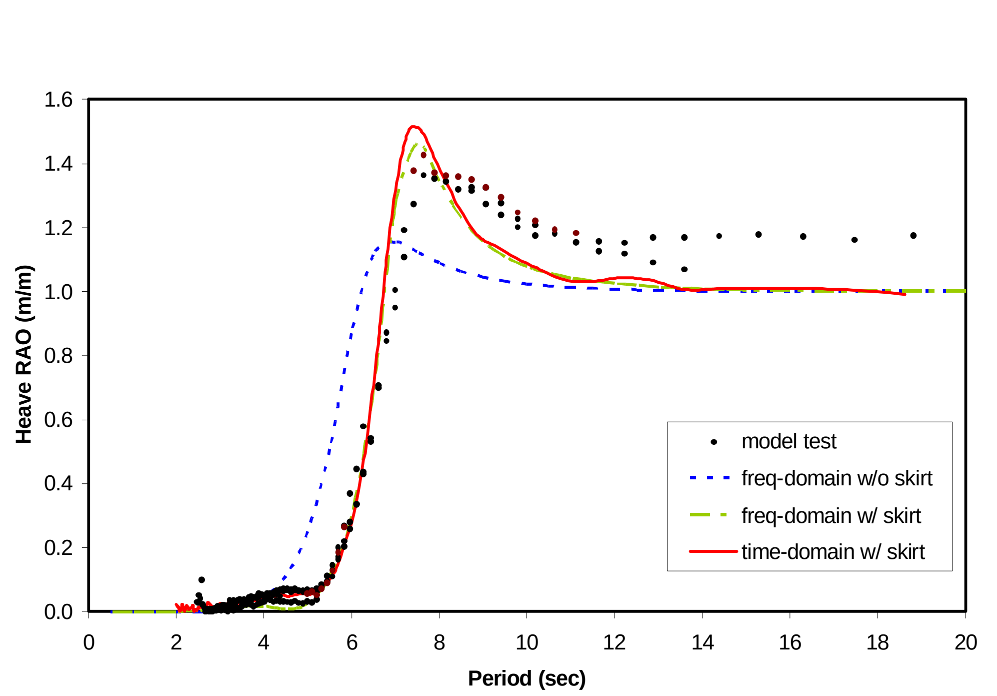
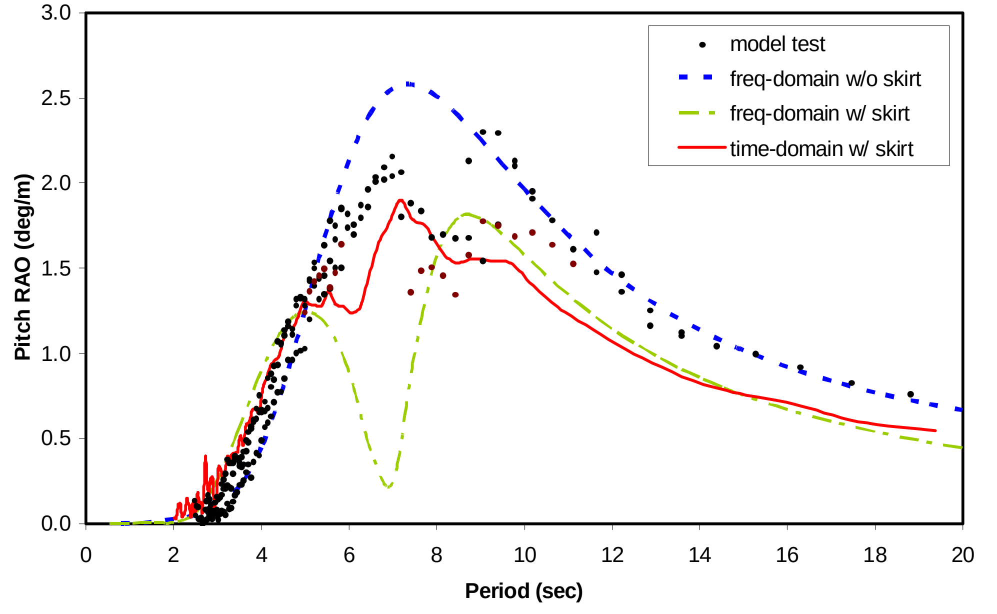
Validation with Model Test Results

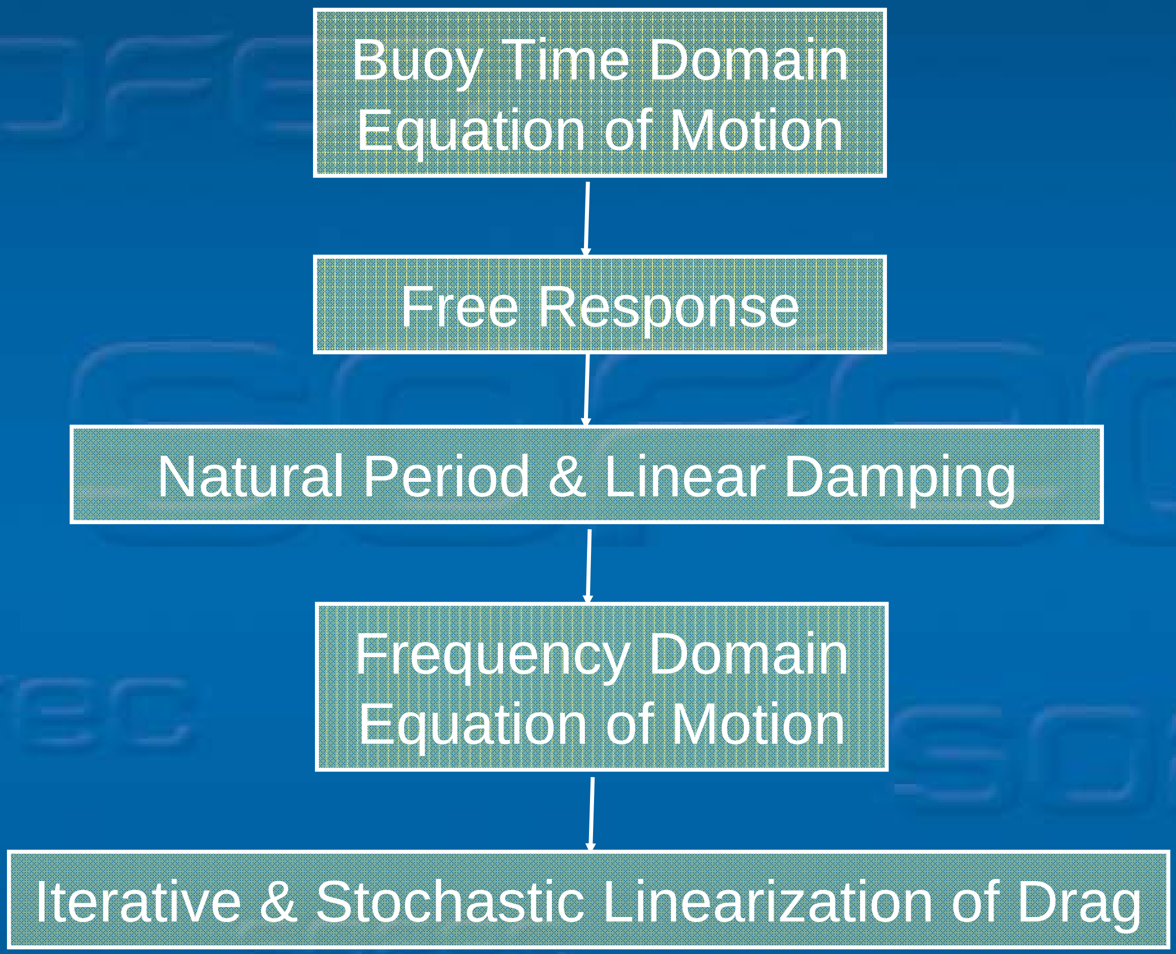






(After Duggal and Ryu 2005)

Pitch motion response



Equation of Motion (6-DOF & 1-DOF)

Spring-Mass-Damper

$$M\{\ddot{x}\} + C\{\dot{x}\} + K\{x\} = \{F\}$$

$$M = (M_i + M_a)$$

$$C = (C_r + C_v)$$

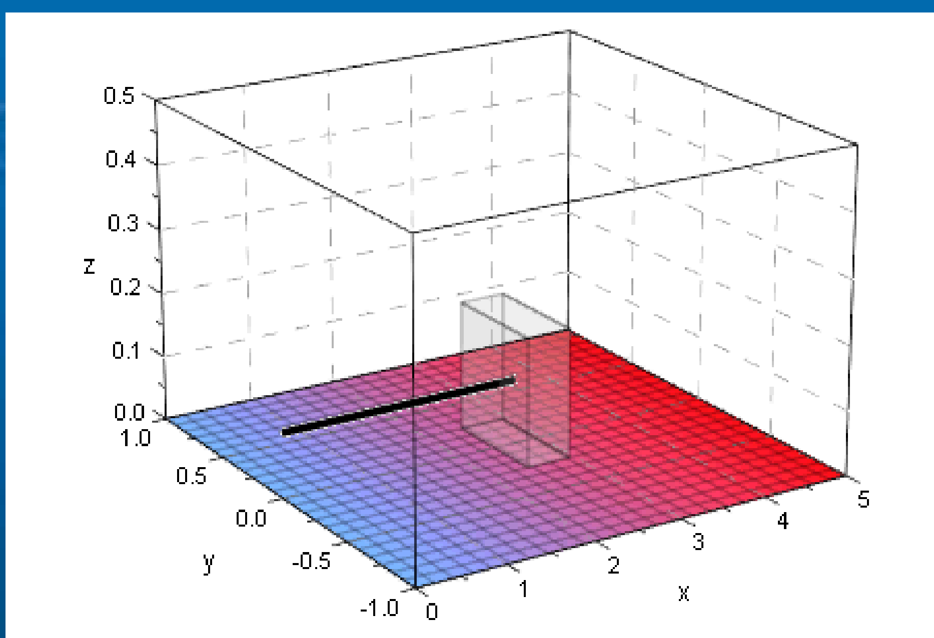
$$K = (K_h + K_m)$$

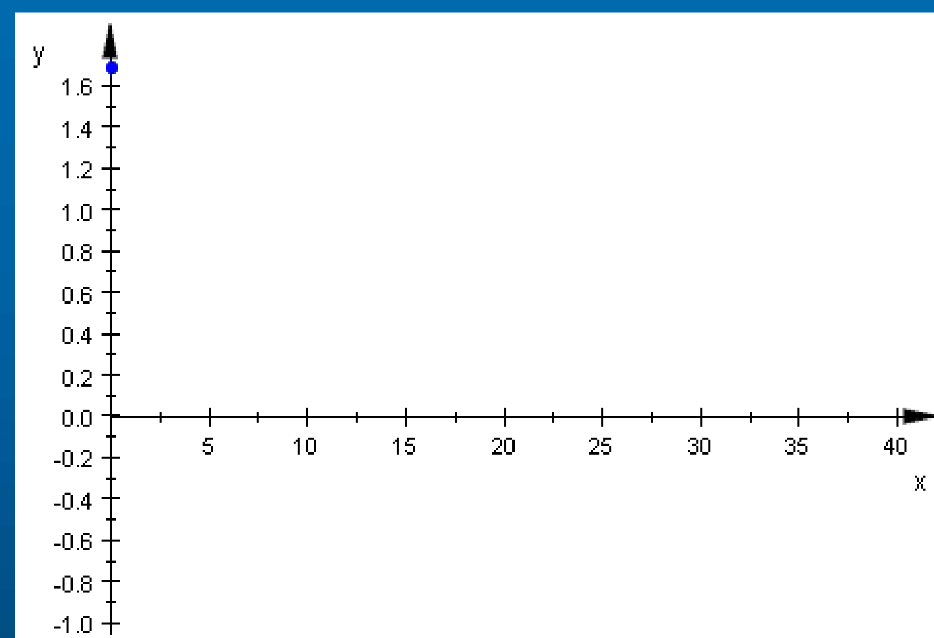
Pitch Free Response

$$M\ddot{x} + C\dot{x} + Kx = 0$$

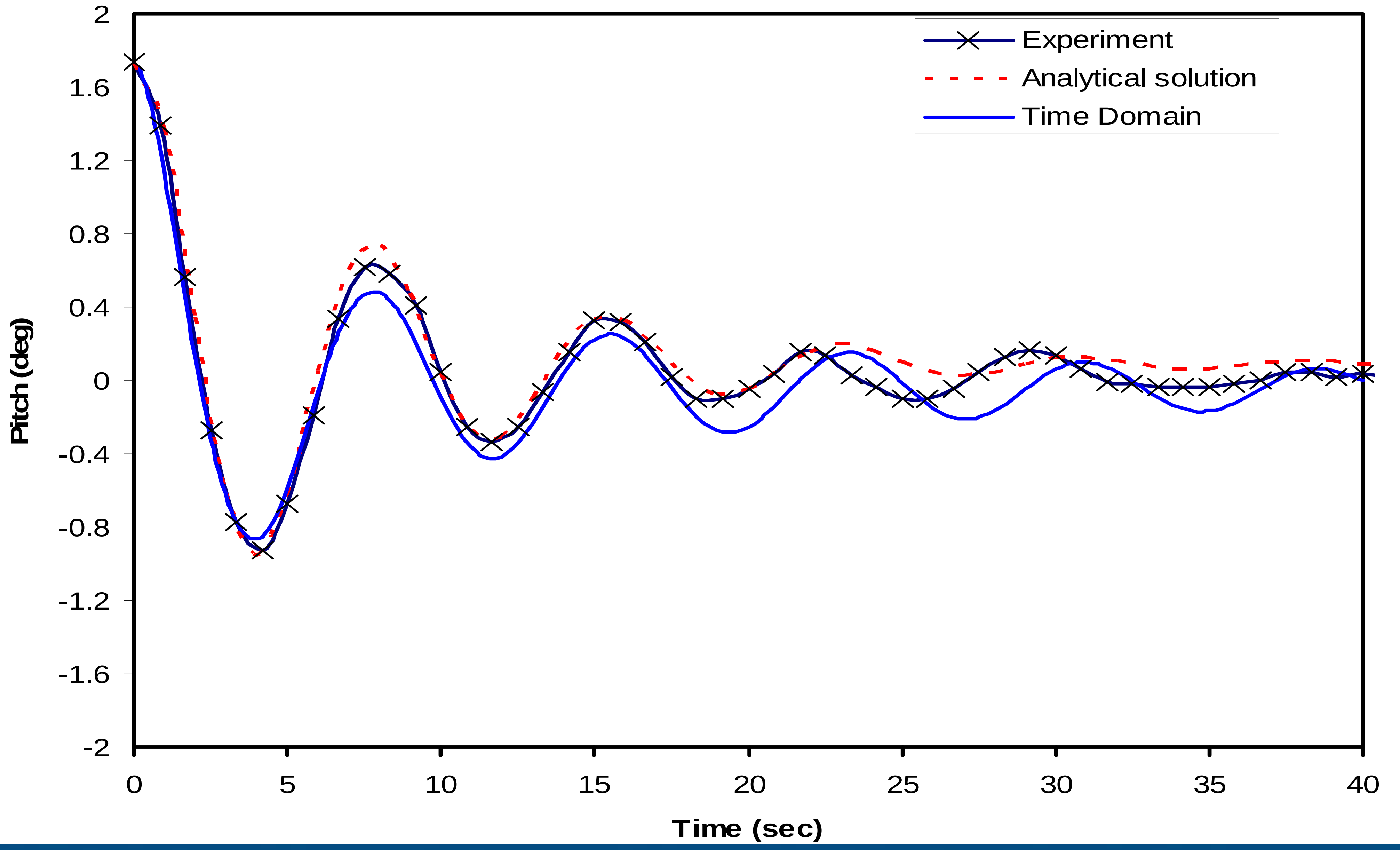
$$x(0) = X_o,\ \dot{x}(0) = V_o$$

$$x_h(t) = X_o e^{-\zeta\omega_o t}\cos\left(\sqrt{1-\zeta^2}\,\omega_o t + \varphi\right)$$

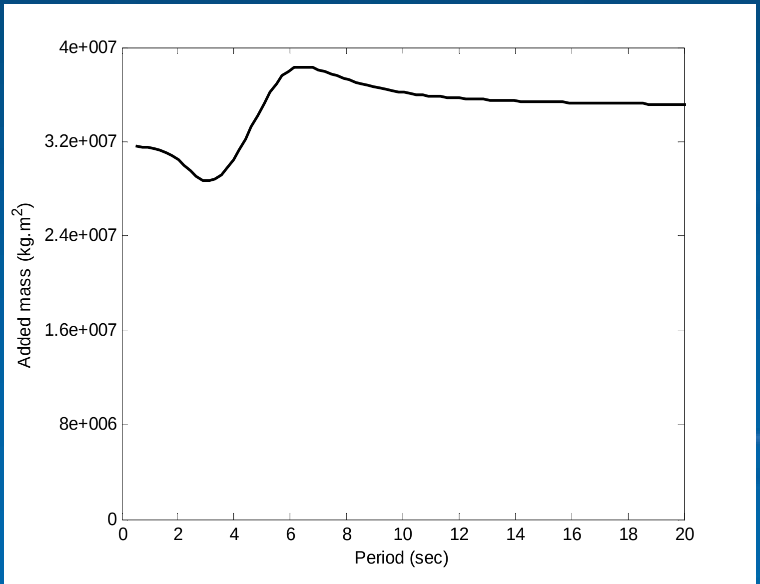


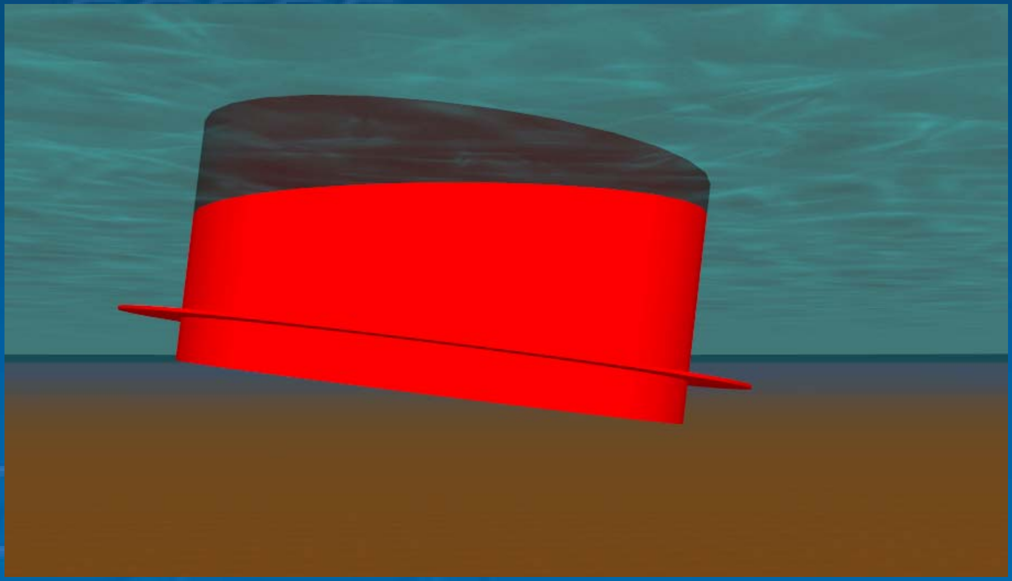


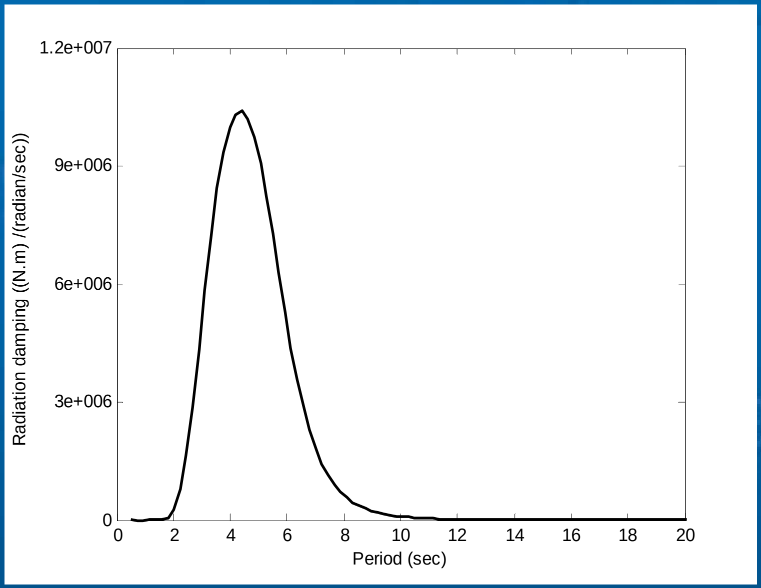
Pitch Free Decay Comparison

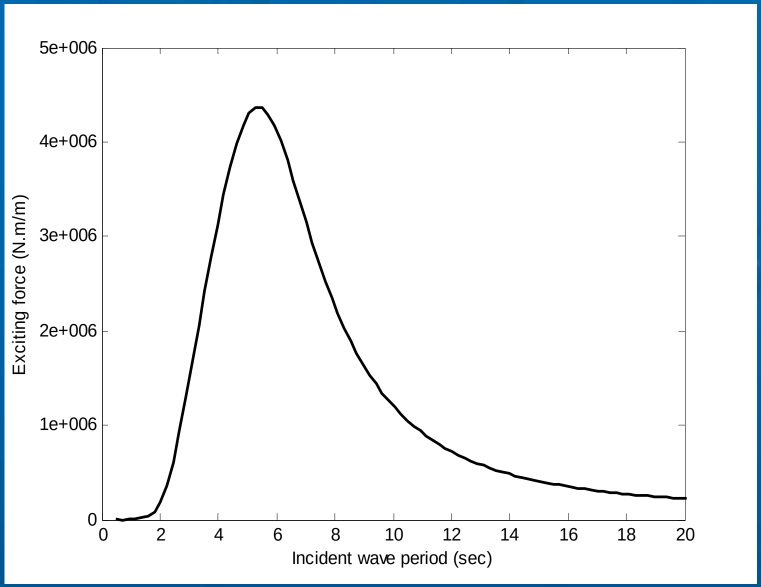


Freq. dependent pitch motion elements









Damping Linearization: 1- Quadratic 2- Cubic

$$M\ddot{x} + C(\dot{x})\dot{x} + Kx = F$$

$$C(\dot{x})\dot{x} = C_1\dot{x} + \frac{8}{3\pi}C_2\omega |x|\dot{x} + \frac{3}{4}C_3\omega^2 x^2\dot{x}$$

(Chakrabarti, 1992)

$$X^0 = 0, C_v^* = \frac{8}{3\pi}C_v\left|x^n\right|, C_v = \frac{1}{2}\rho_W C_D A$$

$$X^{n+1} = \frac{F(\omega)}{-\omega^2 M - \left(C_r + C_v^*\right)i\omega + K}$$

$$\left|X^{n+1} - X^n\right| \leq \varepsilon$$

Damping Linearization: 3- Stochastic

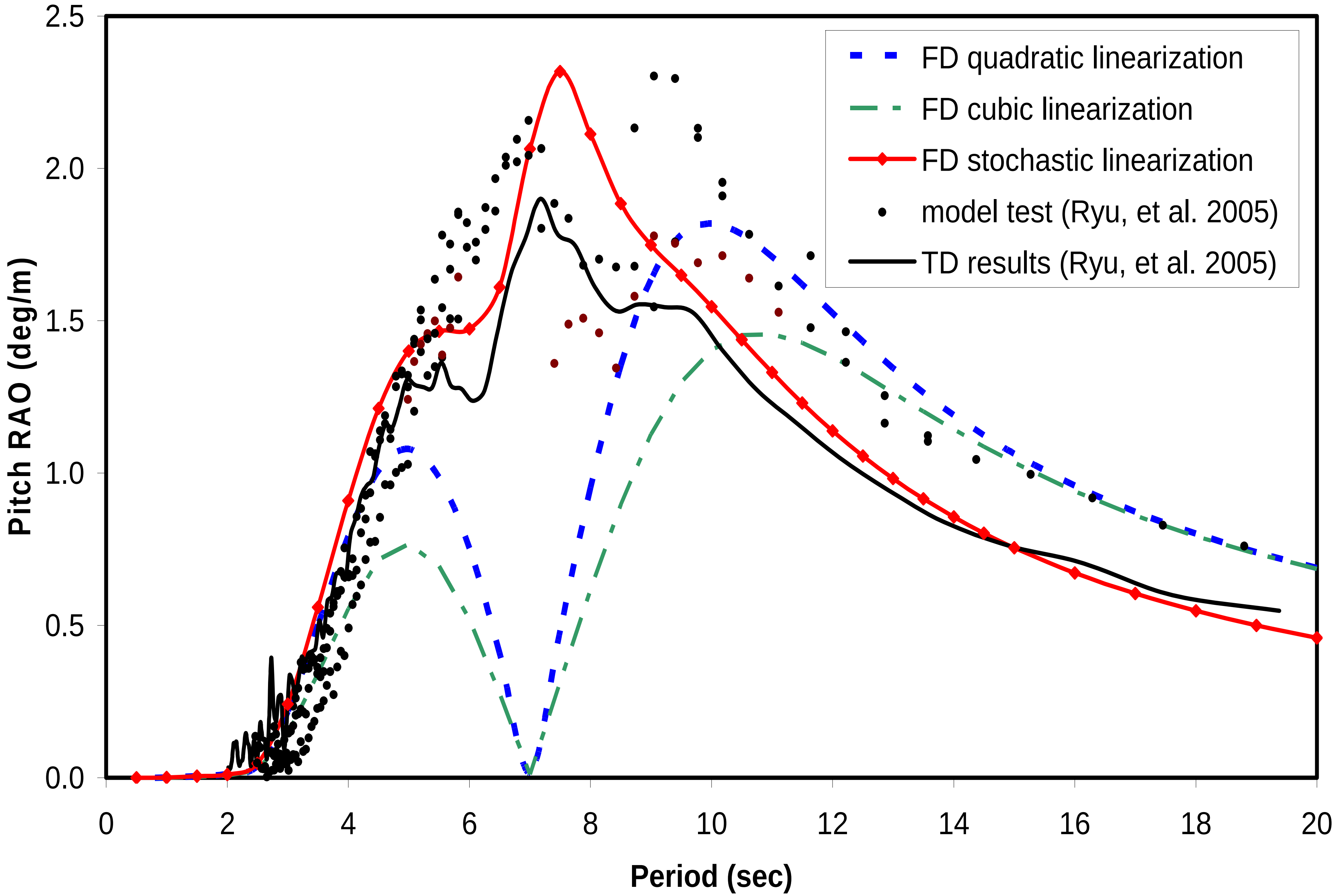
$$M\ddot{x} + C(\dot{x})\dot{x} + Kx = F$$

$$C(\dot{x})\dot{x} = \frac{1}{2}\rho_W C_D A\sqrt{\frac{8}{\pi}}\sigma_{\dot{x}}\dot{x}$$

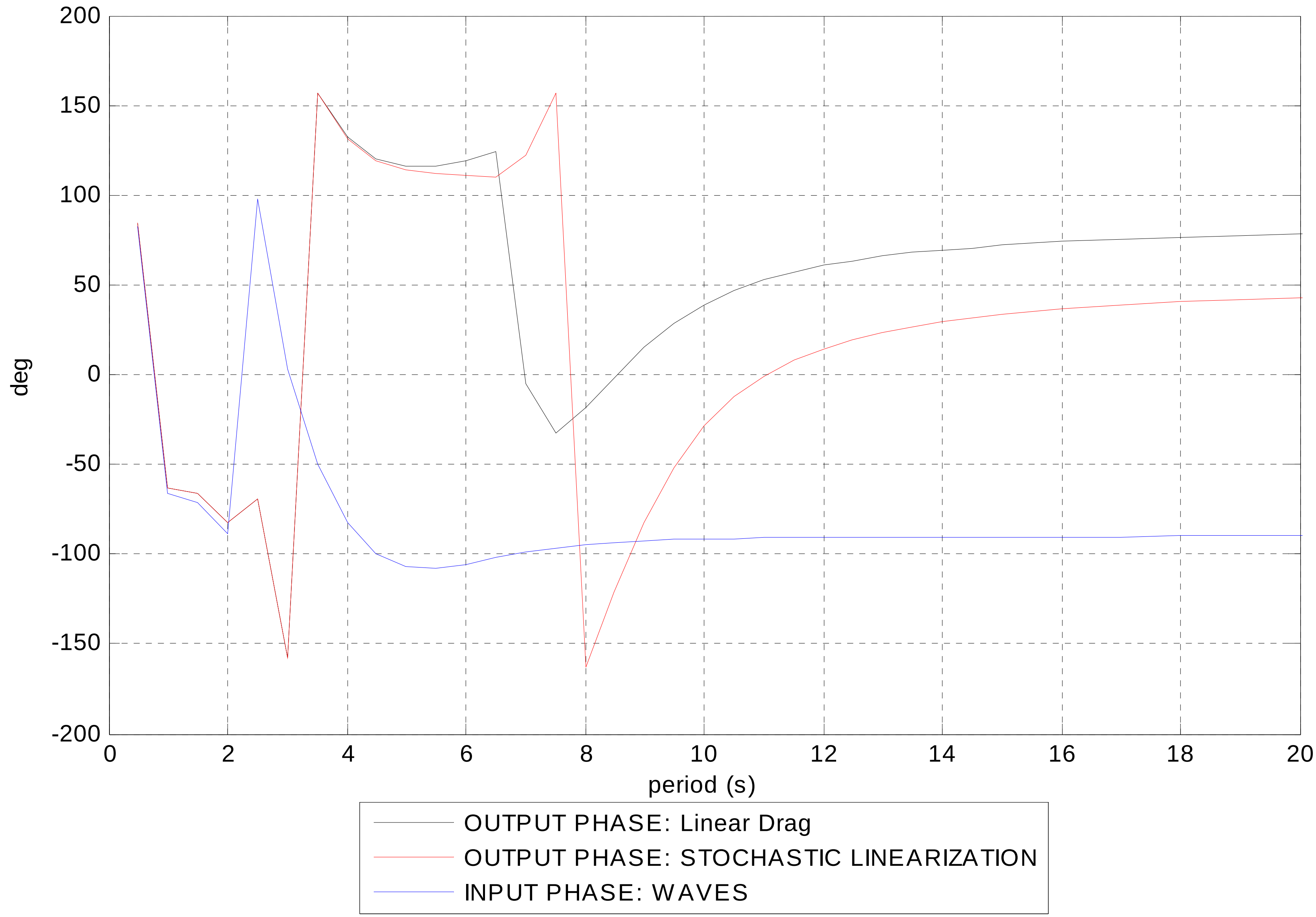
(Chakrabarti, 2002)

$$X(\omega) = \frac{F(\omega)}{-\omega^2 M - \left(C\sqrt{\frac{8}{\pi}}\sigma_{\dot{x}}\dot{x}\right) + K}$$

Pitch Motion RAO Comparison



Phase Comparison



Summary

- Linearization quadratic drag/damping is accomplished by quadratic, cubic and stochastic linearizations.

- Quadratic and cubic linearization shows the deep valley around 6 to 8 seconds & good agreement for long periods.

- Stochastic linearization show a good agreement.

Thank You