Frequency-Domain Calculations of Moored Vessel Motion Including Low Frequency Effect

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Outline

1. Background
2. Summary of TD and FD Analyses
3. Case Studies
4. Comments on Modeling
5. Summary

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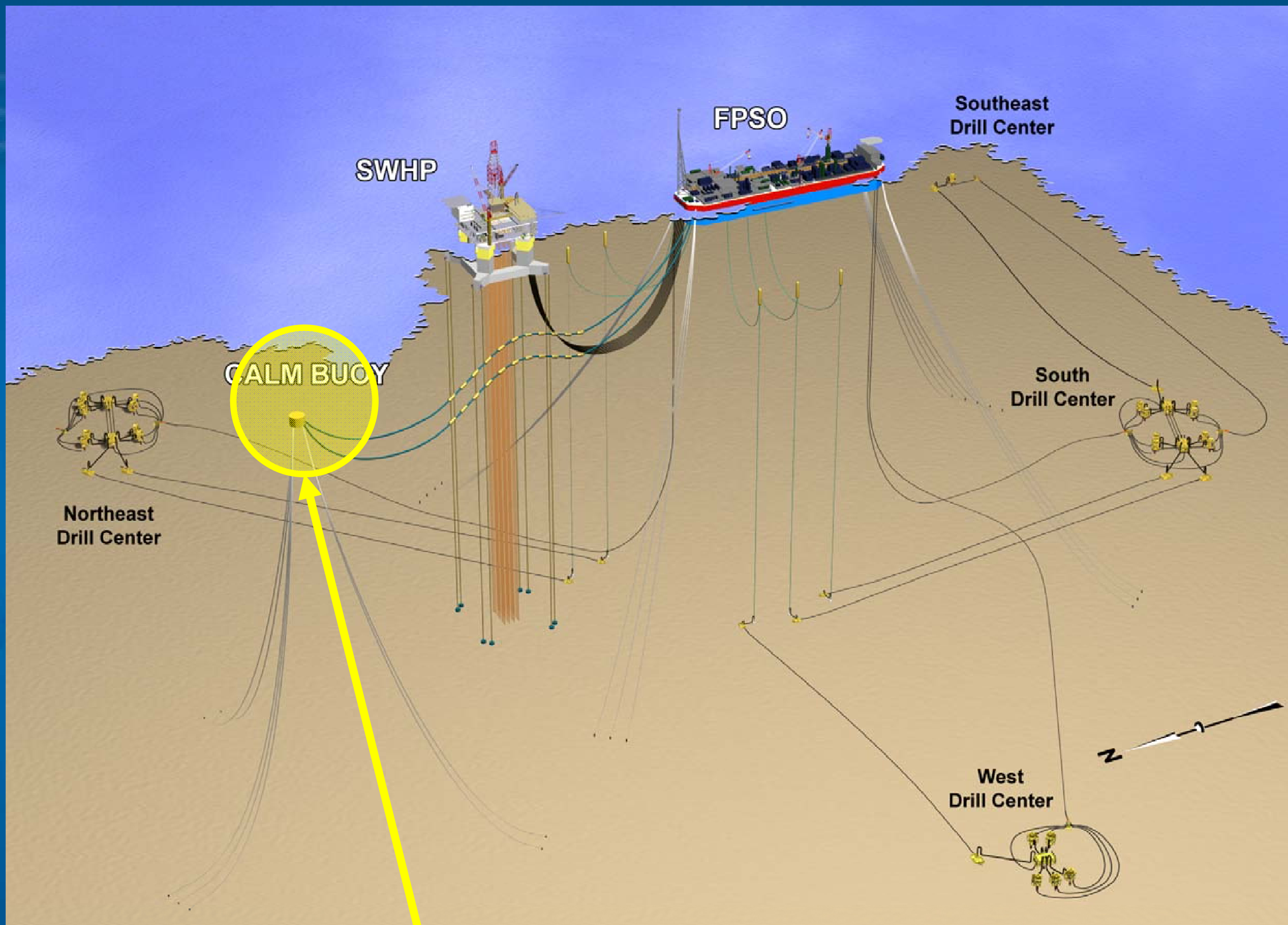
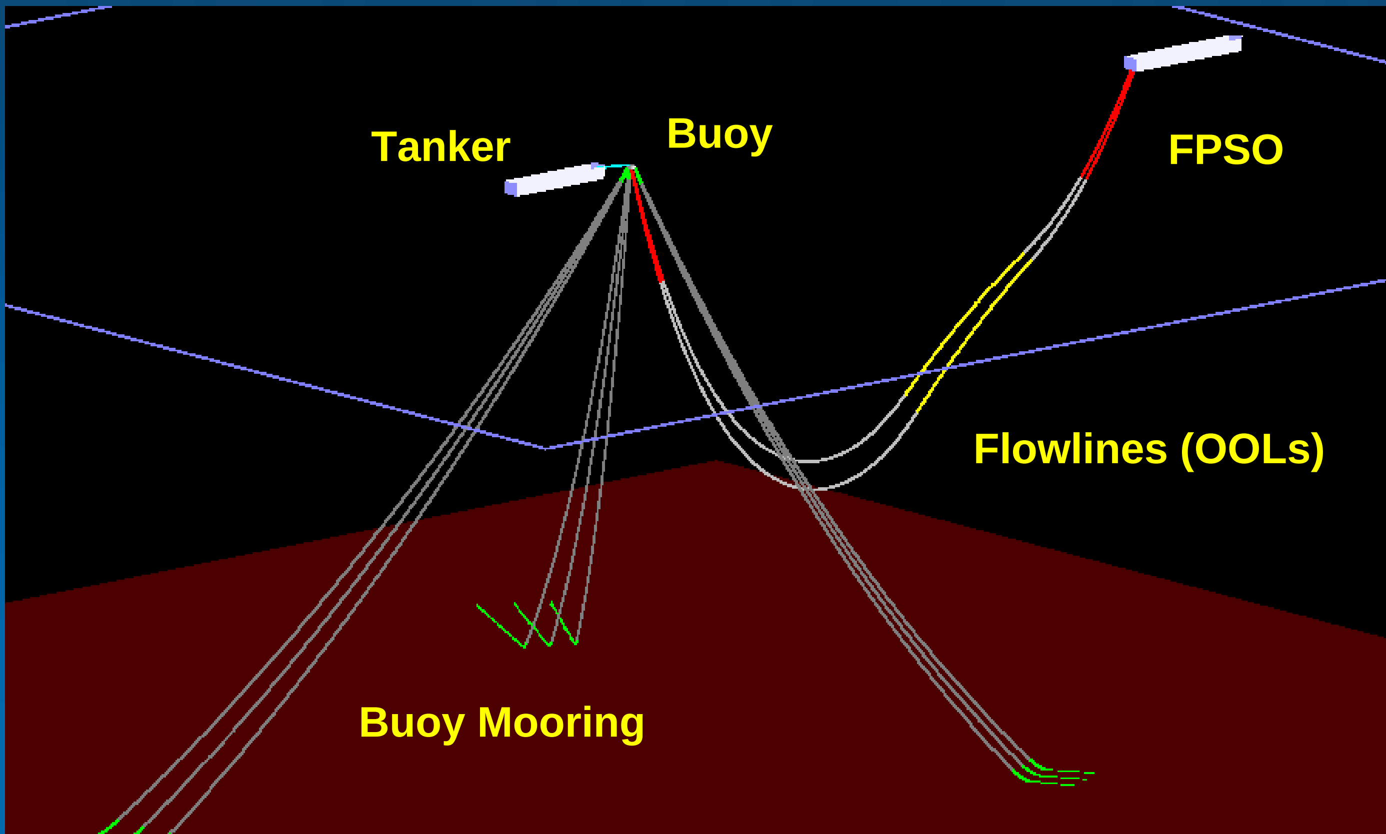
Background

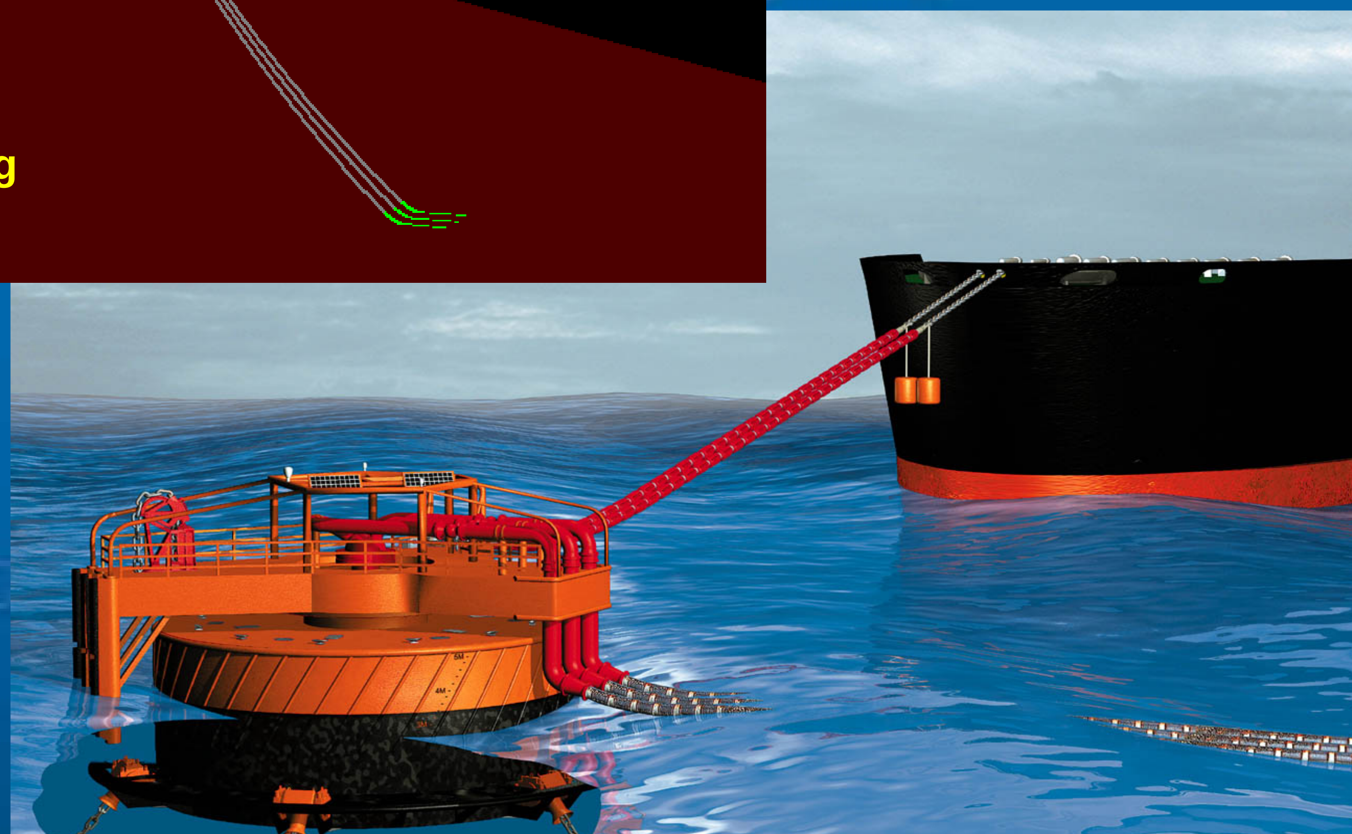
"Derivation of CALM Buoy Coupled Buoy RAOs in Frequency Domain and Experimental Validation" (Le Cunff *et al*., 2007)



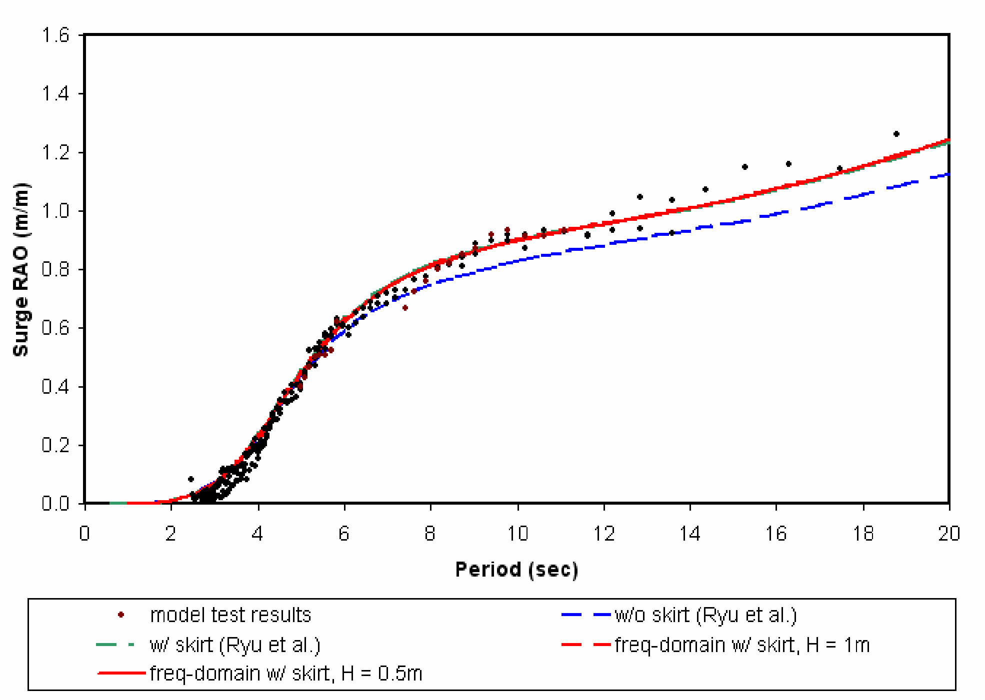
deepwater CALM buoy

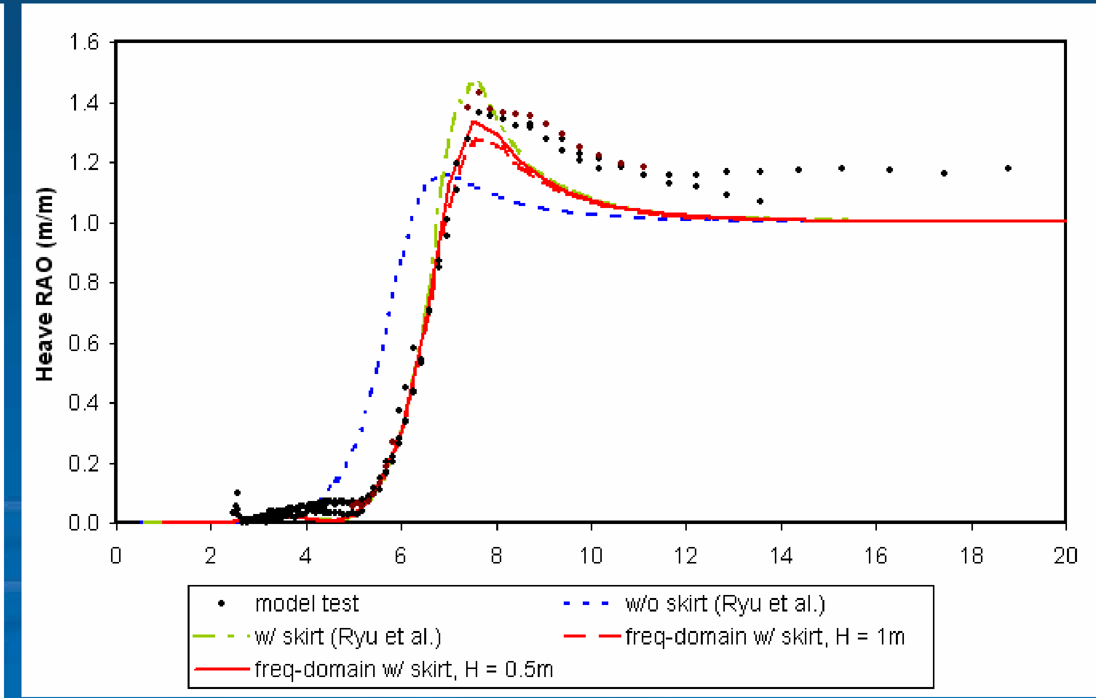
CALM Buoy System with Tanker Connected

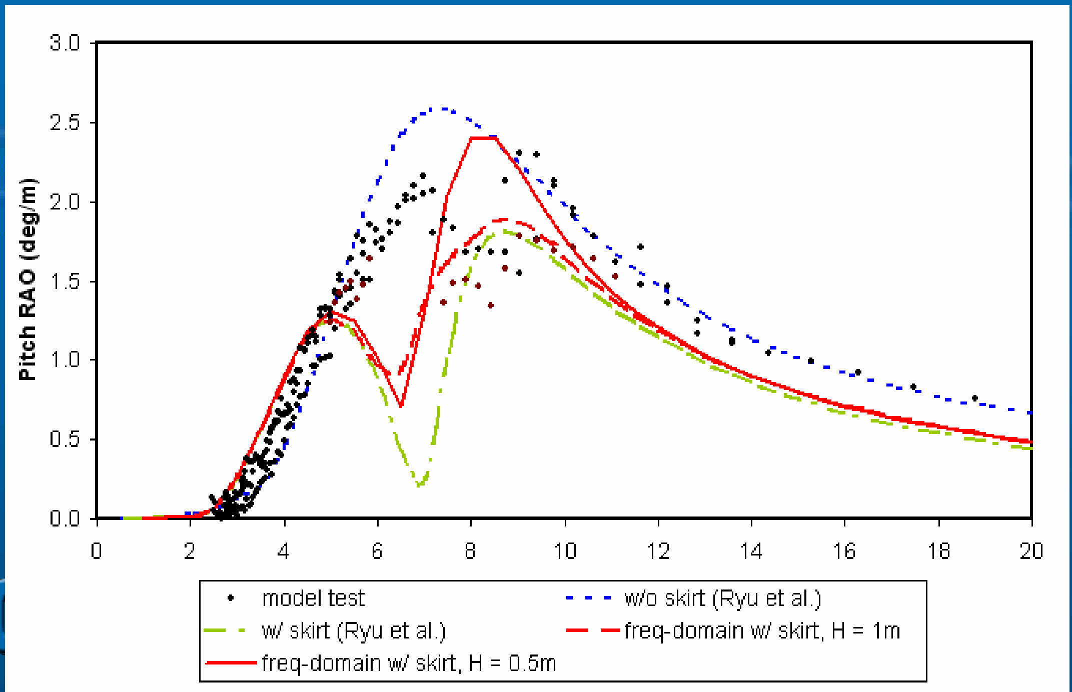




Validation with Model Test Results







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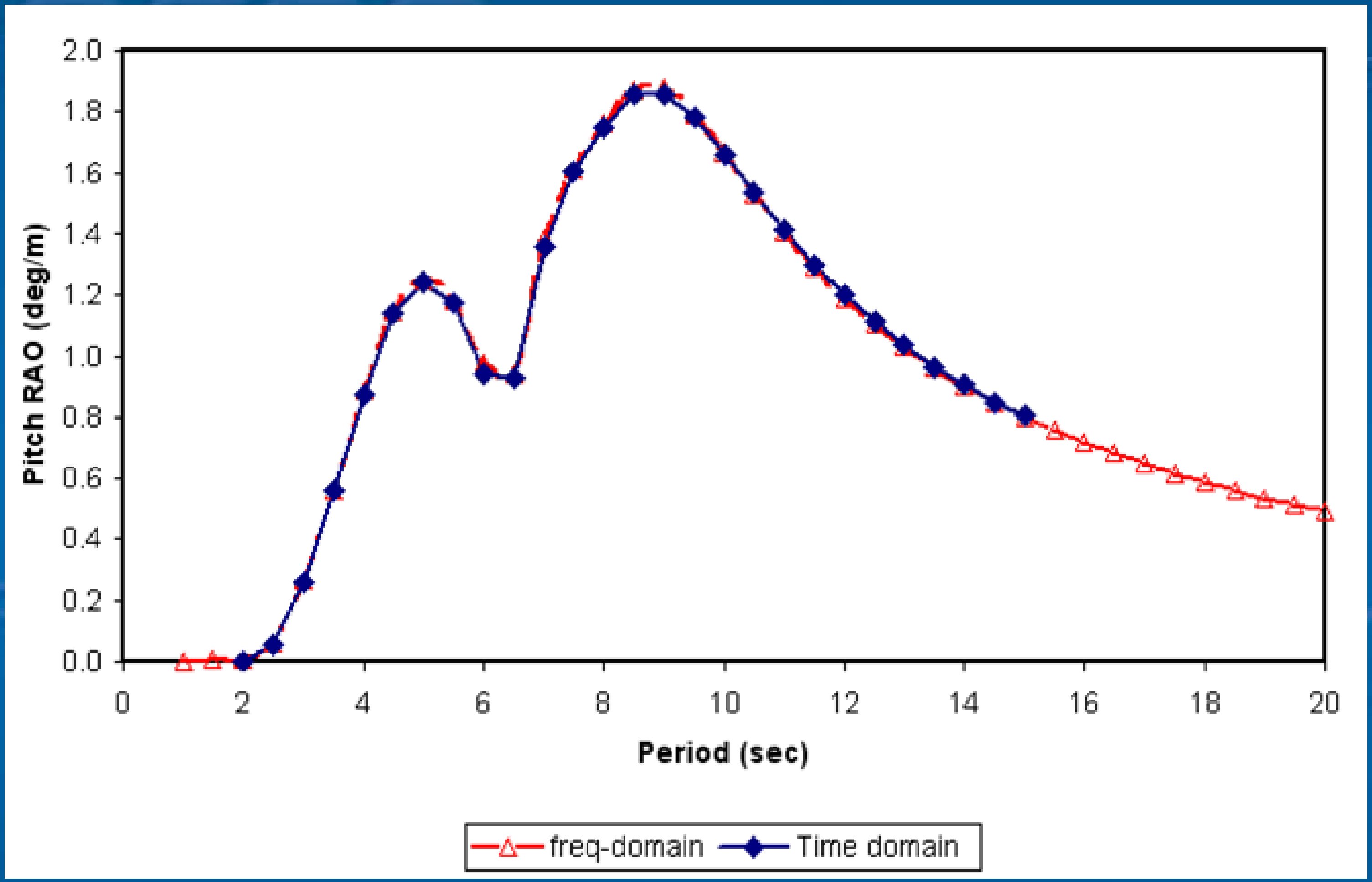
Comparisons between TD and FD (Pitch Motion)



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Time-Domain Analysis

- Hydrodynamic Loads on the Bodies
  - hydrodynamic calculations via BEM method
  - added mass, radiation damping, first order wave force
- Loads on the Lines
  - FEM-based computer program (DeepLines)
  - lines characteristics + Morison's formulation
- Coupled System
  - extra node with six DOF for the floating body
  - solve the coupled equation:

$$\left\{M + M_a(\infty)\right\} \ddot{\vec{X}} + \int_0^\infty R(t-\tau)\dot{\vec{X}} d\tau + K\vec{X} = \vec{F}_d + \vec{F}_w^{(1)} + \vec{F}_w^{(2)} + \vec{F}_{line}$$

Frequency-Domain Analysis

- Equation of motion: $$Kx + B\dot{x} + M\ddot{x} = F$$
- Assuming that the position at time *t* is given by:

with $$X(t) = X_{stat} + \sum_{i=1}^{n}\left\{\mathrm{Re}\left\{\left[\sum_{imp=1}^{Nimp} a_{imp}(\omega_i)x_{imp} + x_f(\omega_i)\right]e^{-j(\omega_i t)}\right\}\right\}$$

  – Static equilibrium : $$K_{stat}x_{stat} = F_{stat}$$

  – Imposed displacement : $$K_{stat}x_{imp} = 0$$

(cont'd)

- Loads on body from hydrodynamic calculations
- Dependency of matrices on frequency

$$\left[K(\omega) - j\omega\left(B + B_a(\omega)\right) - \omega^2\left(M + M_a(\omega)\right)\right]\left\{X(\omega)\right\} = \left\{F(\omega)\right\}$$

- Linearization of quadratic viscous damping (both body and lines)

$$|v_{rel}|v_{rel} \approx \Omega(v_{rel})v_{rel}$$

  – Linearization Coefficients
    - Regular waves : $\Omega = \frac{8}{3\pi}A$ (A: norm of velocity)
    - Irregular waves: $\Omega = \sqrt{\frac{2}{\pi}}2\sigma$ (σ: standard deviation)

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Low-Frequency Wave Loading

QTF:

- **Direct calculation**

2) Newman's approximation

- wave elevation:

$$\eta = \mathrm{Re}\left(\sum_{l=1}^{n} A_l \exp\left[-j\omega_l t + j\theta_l\right]\right)$$

- low-frequency (LF) force, Molin (2002)

$$F_{wave}^{(2)} = \mathrm{Re}\left(\sum_{l=1}^{n}\sum_{m=1}^{n} A_l A_m f^{(2)}(\omega_l, \omega_m) \exp\left[-j(\omega_l - \omega_m)t + j(\theta_l - \theta_m)\right]\right)$$

- LF force spectrum

$$S_F^{(2)}(\Omega) = 8\int_0^{\infty} S(\omega) S(\omega + \Omega) \left| f^{(2)}(\omega, \Omega - \omega) \right|^2 d\omega$$

Low-Frequency Wave Loading (QTF)

- Newman's approximation

$$\left| f^{(2)}(\omega_l, \omega_m) \right|^2 = \left| f_d(\omega_l) \right| \times \left| f_d(\omega_m) \right|$$

- Second order wave potential (x-direction for example)

$$\left| f_x^{(2)}(\omega_l, \omega_m) \right| = \rho (1 + Cm_x) V \frac{\omega_l \omega_m (\omega_l - \omega_m)^2 (k_l - k_m)}{g(k_l - k_m) th\left[(k_l - k_m)h\right] - (\omega_l - \omega_m)^2}$$

$$\times \frac{ch\left[\left(k_l - k_j\right)(z + h)\right]}{ch\left[\left(k_l - k_j\right)h\right]} \cos\beta$$

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Wind Loading

- Wind speed driven from wind spectrum:

$$V_{wind} = \mathrm{Re}\left(\sum_{l=1}^{n} A_l \exp\left[-j\omega_l t + j\theta_l \right] \right)$$

- Wind force:

$$F_{x,wind} = \frac{1}{2} \rho_{air} S C_x \left| V_{wind} - V_{vessel} \right|^2$$

- Evaluation of wind coefficients based on mean direction (wind w.r.t. body)

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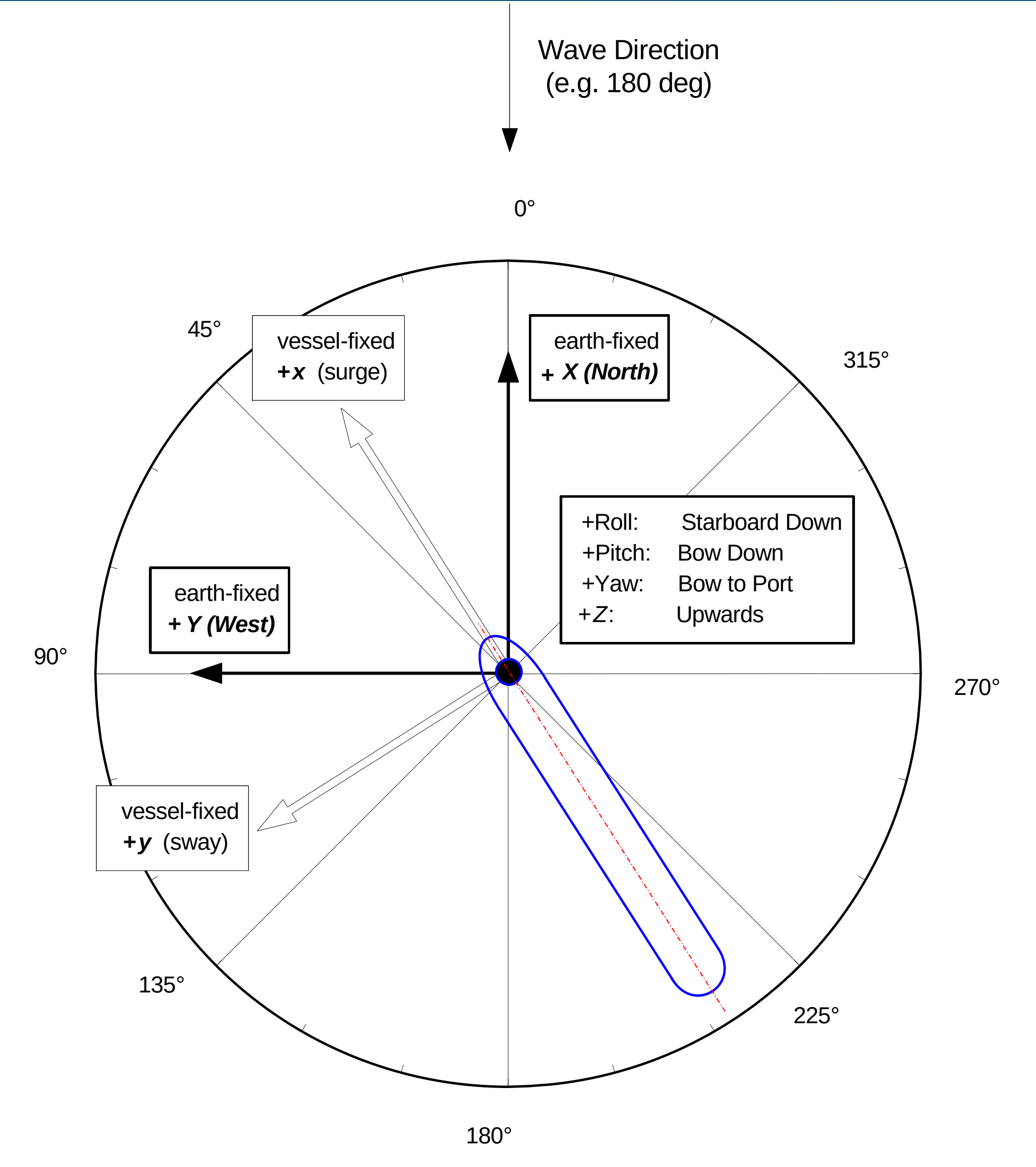
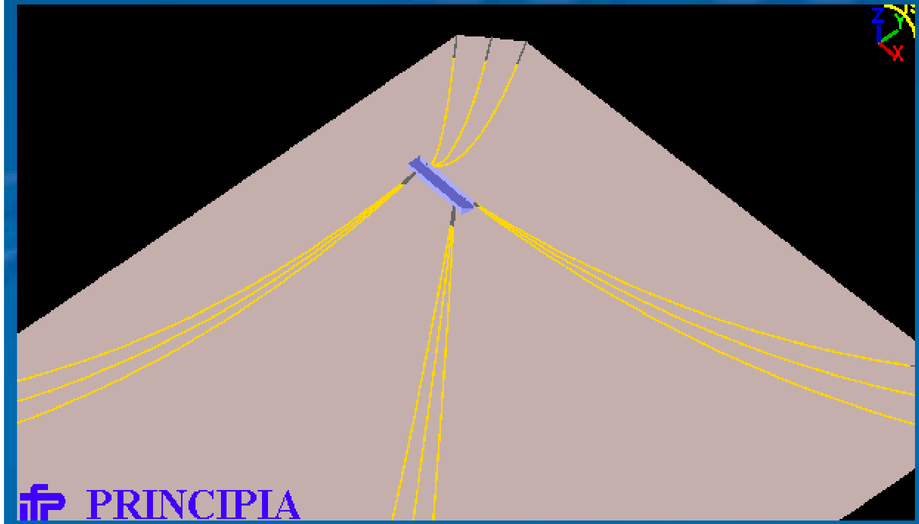
FD/TD Calculation Results Comparison





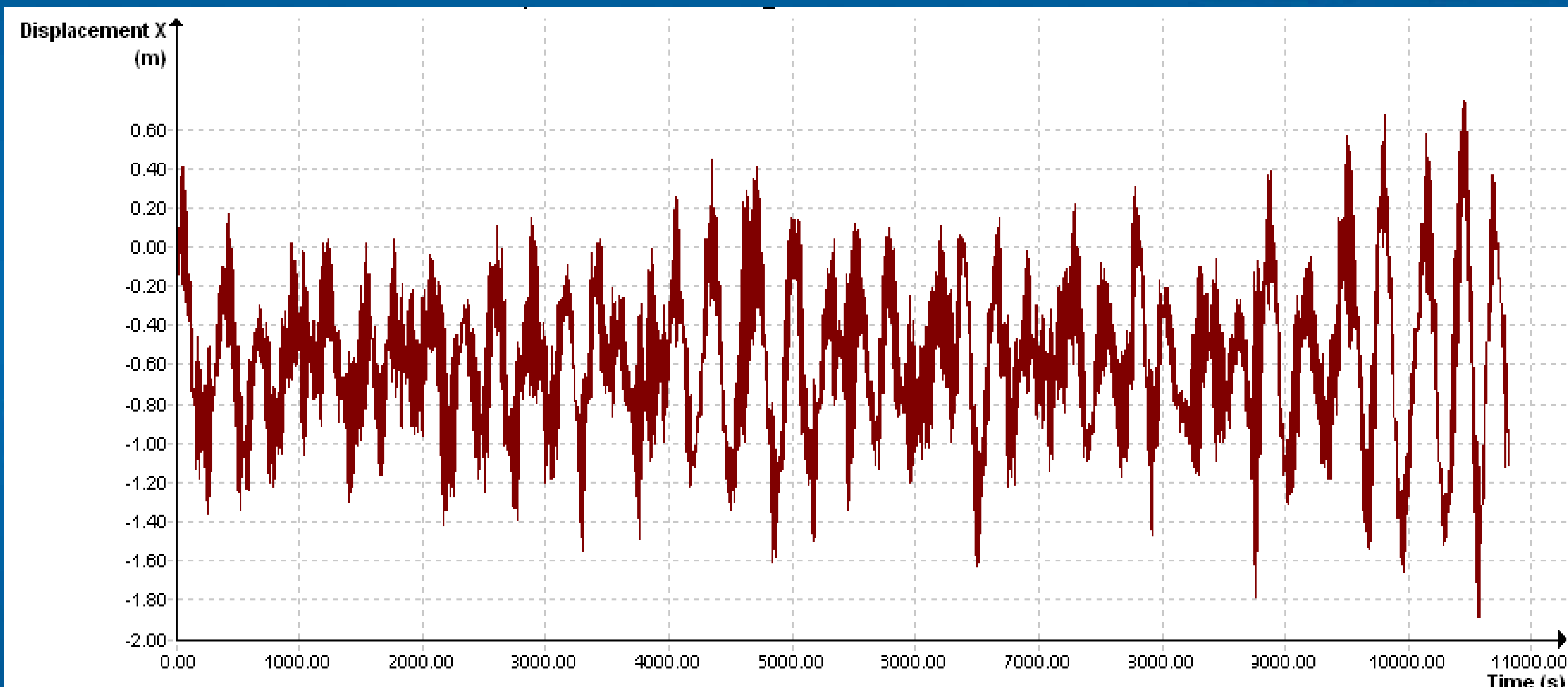
Spread Moored FPSO

Comparison FD/TD:

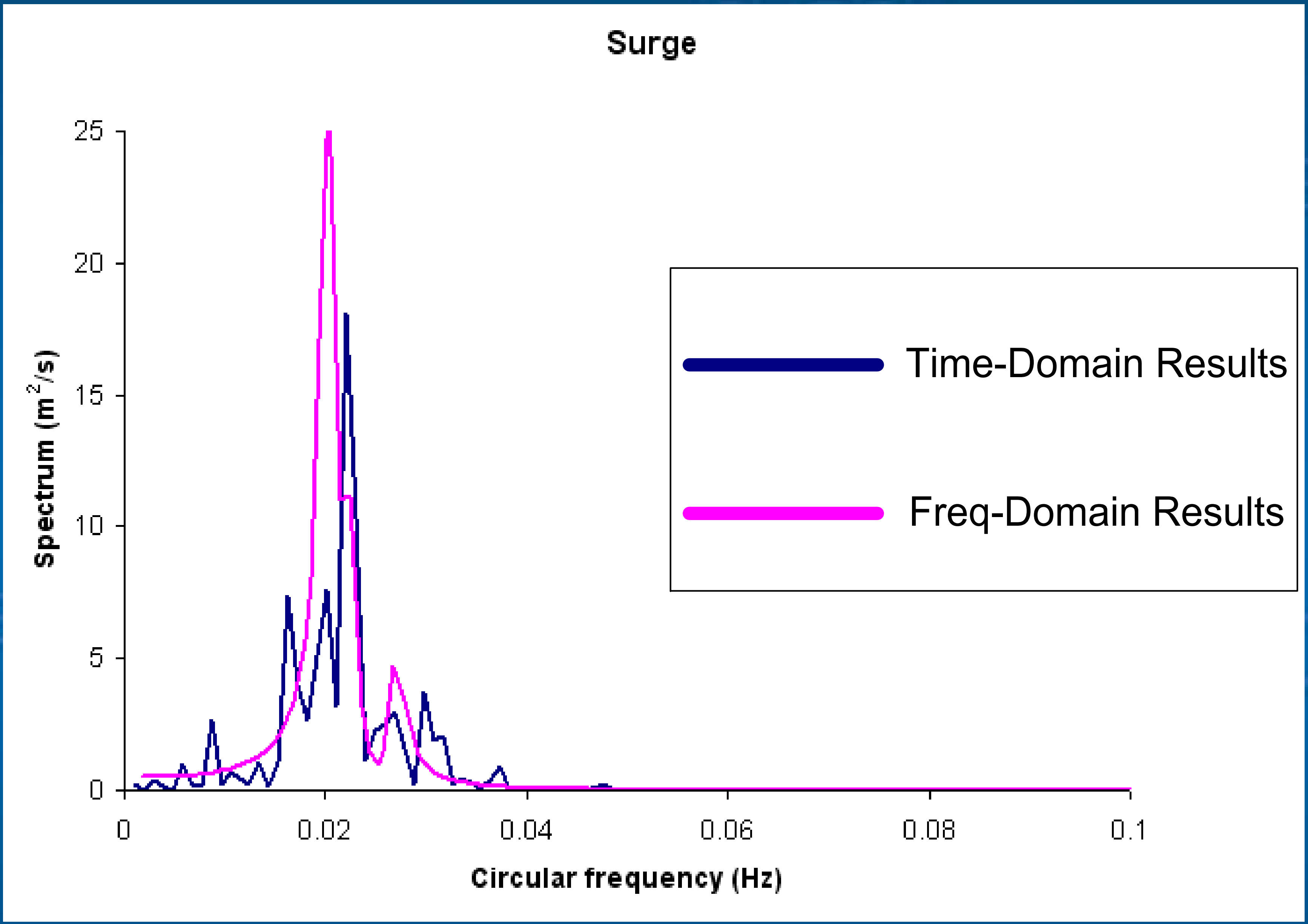
- **FD - spectrum directly computed**
- **TD - FFT of time series**

Time-Domain Calculation

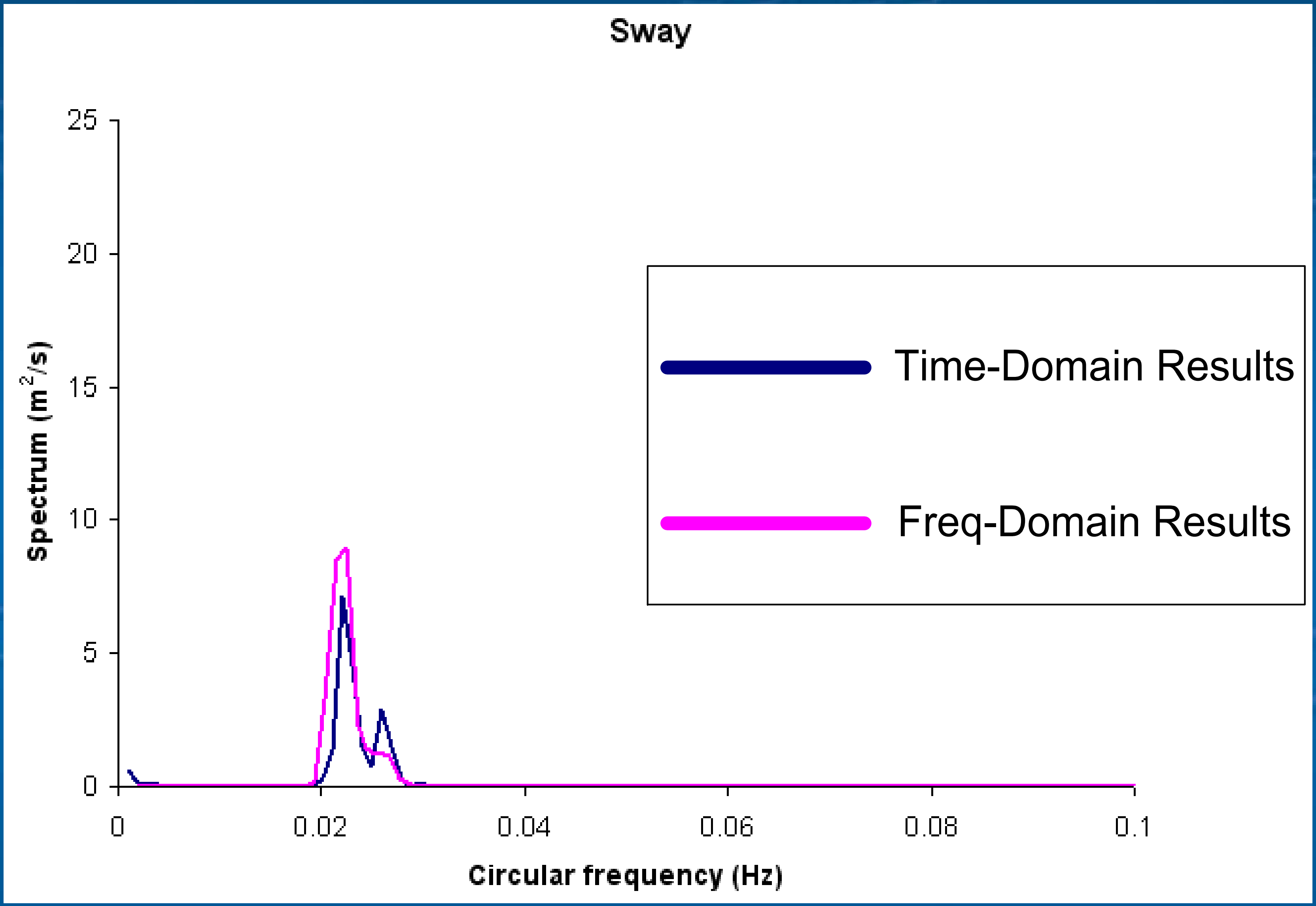
- 3-hour simulation
- LF and WF components



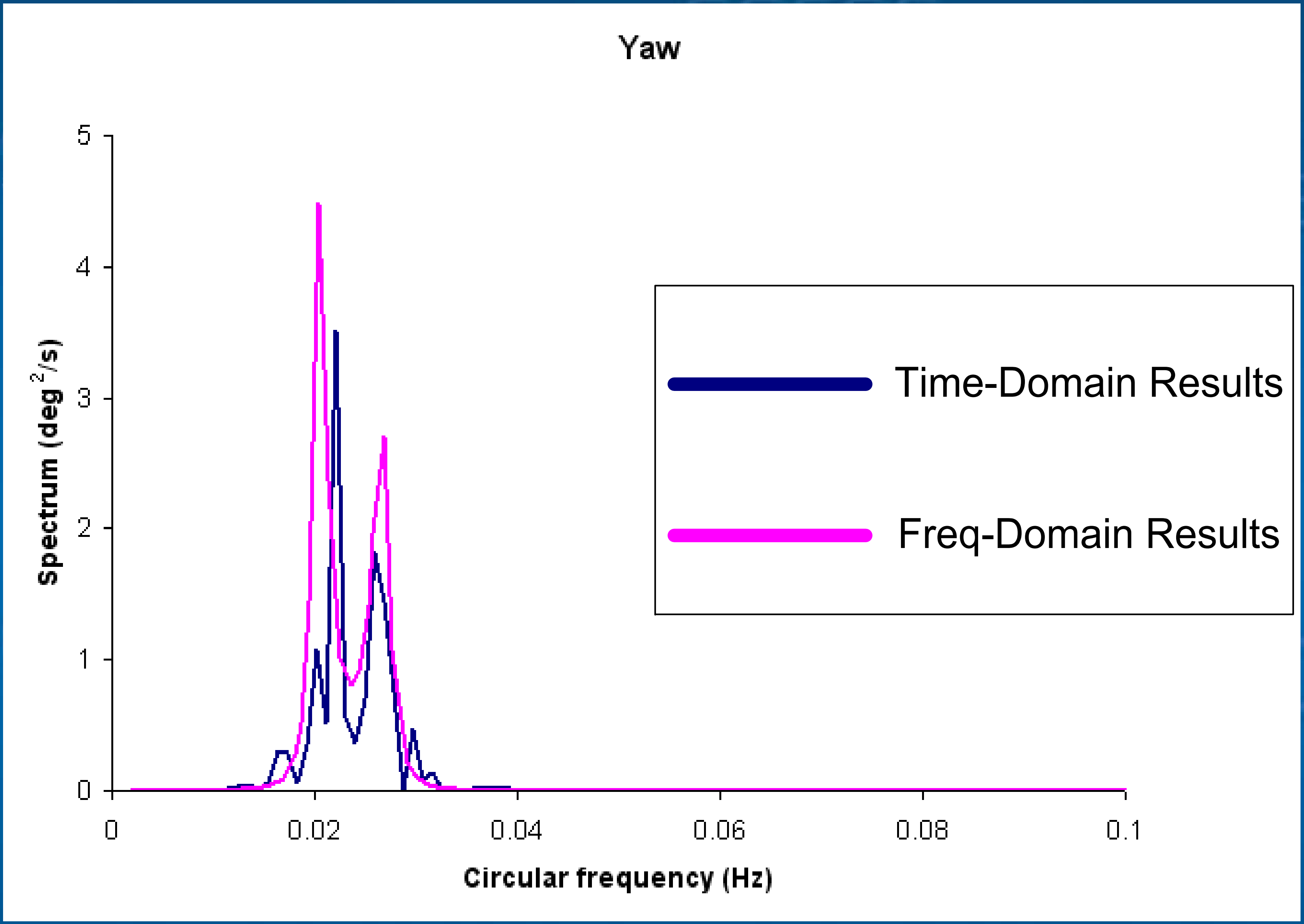
Case 1 (waves only; no wind)
Results – FPSO Surge



Results – FPSO Sway (cont’d)



Results – FPSO Yaw (cont'd)



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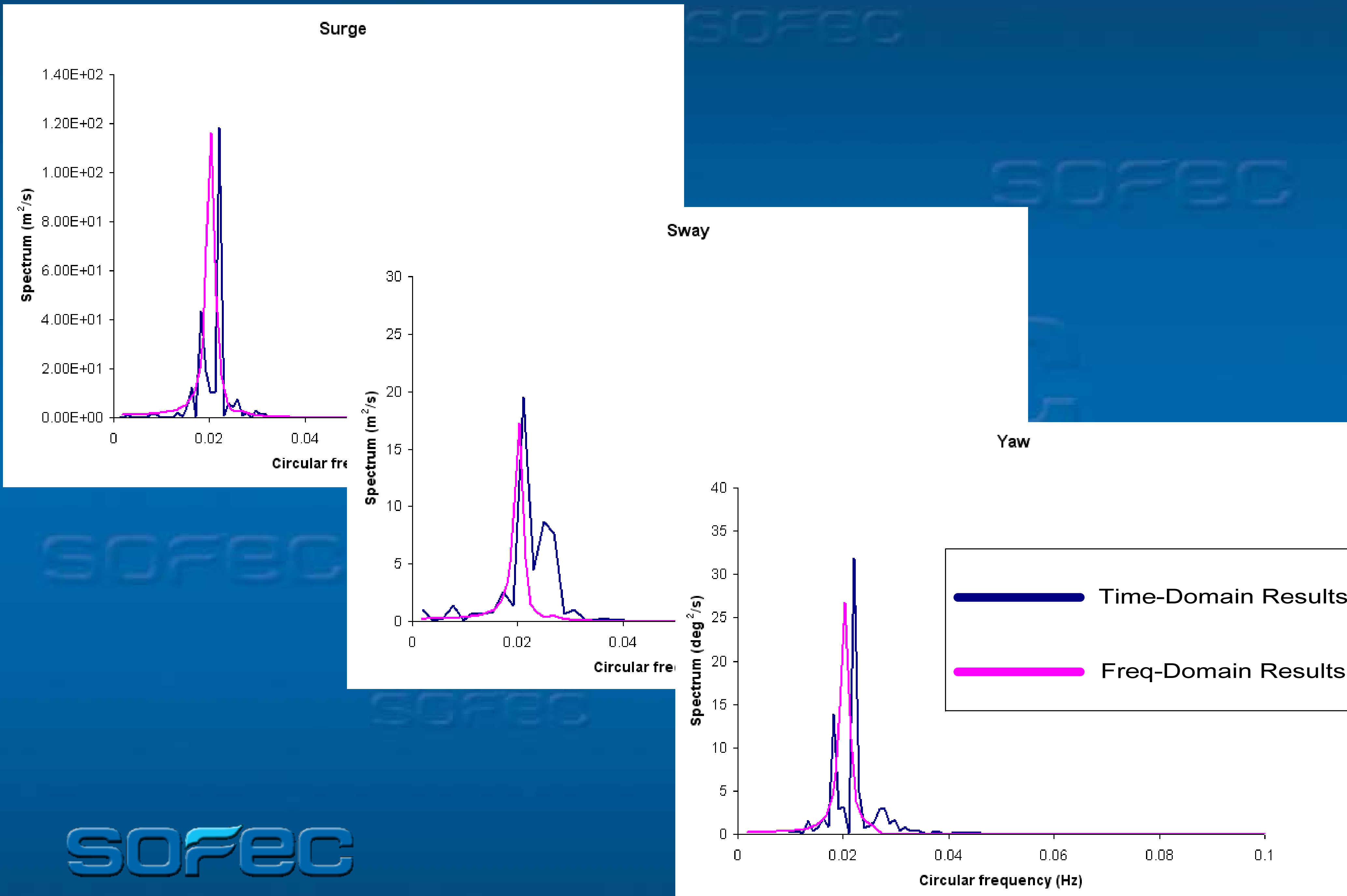
Case 2 (waves + wind)
Results – FPSO LF Motions



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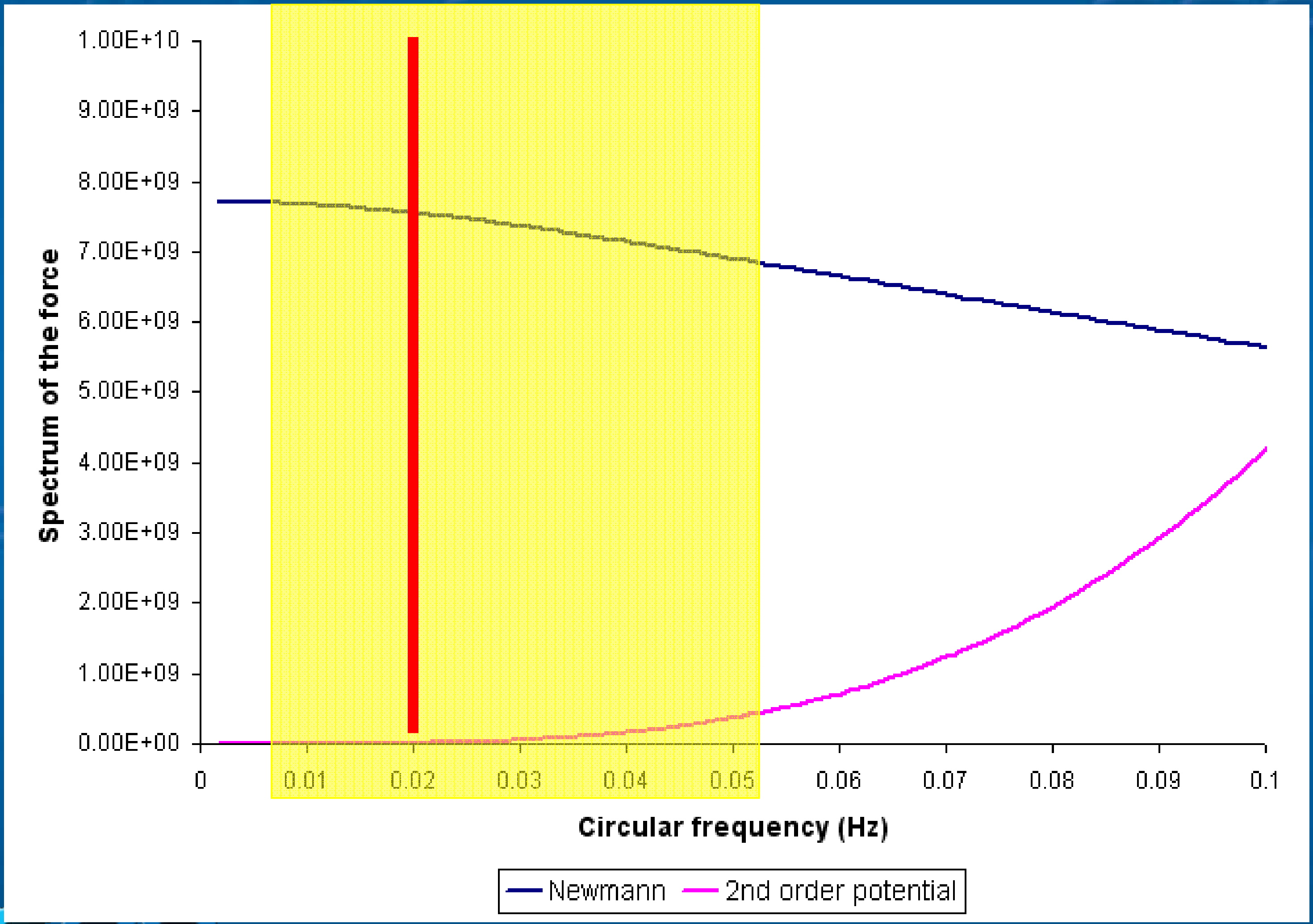
1. Second-Order Wave Potential

- Effect negligible
- Induces a very small force spectrum at low frequency range



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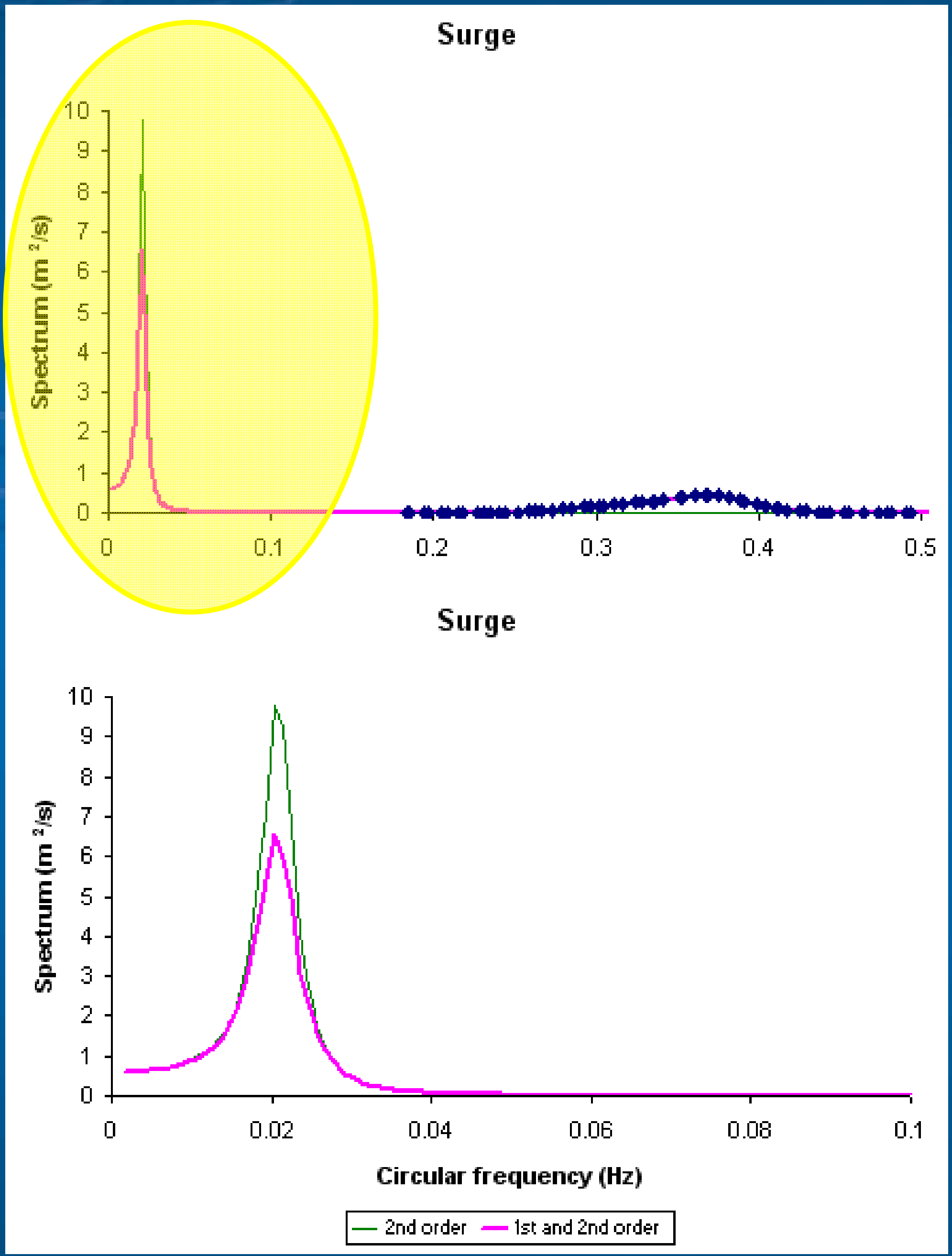
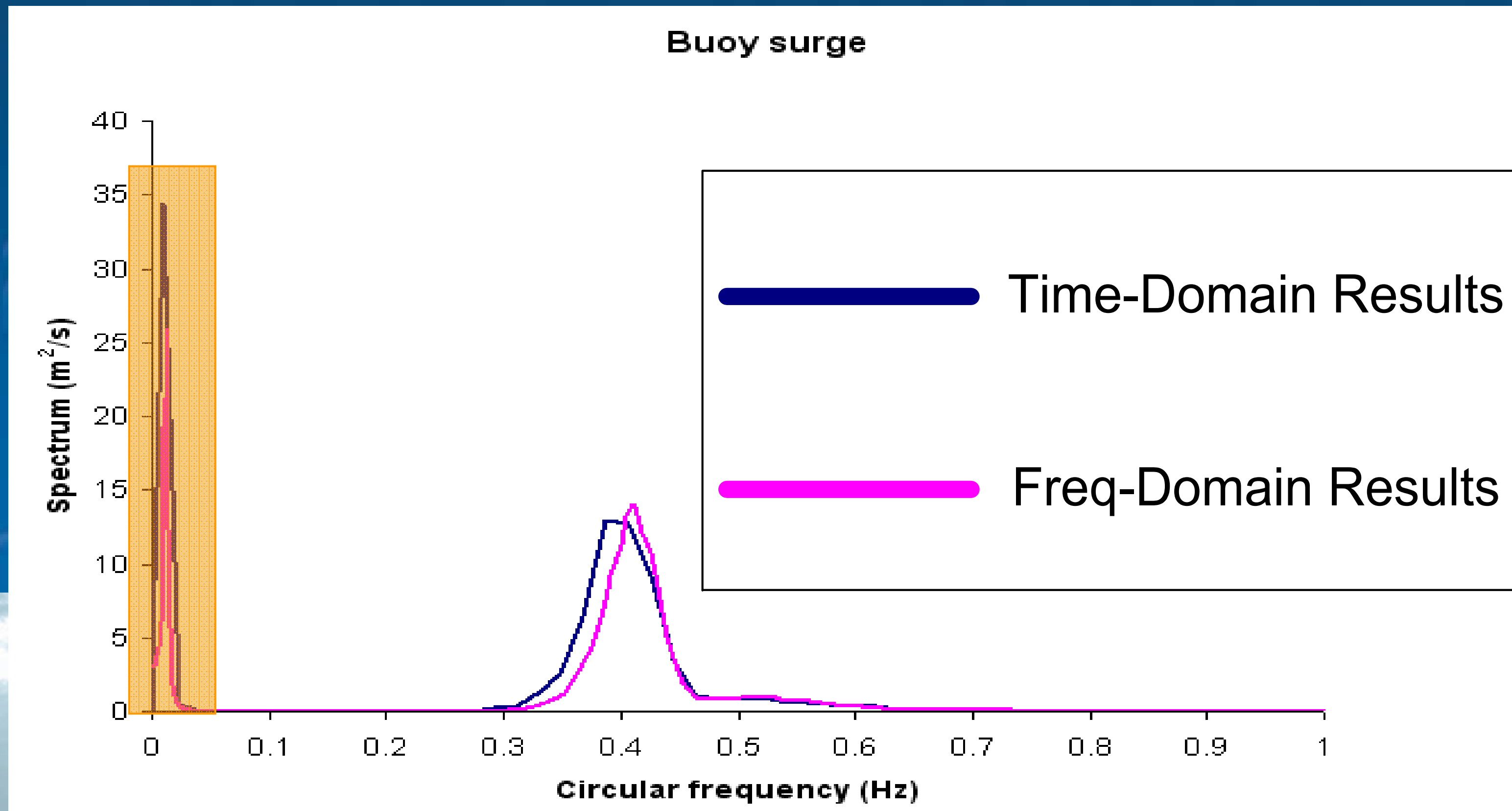
2. Reduction of LF Contribution by WF Components

Comparisons

- first order waves (WF) only
- low frequency (LF) only
- LF + WF



3. Low Stiffness Mooring



- Wave + wind aligned: OK
- Otherwise TD required

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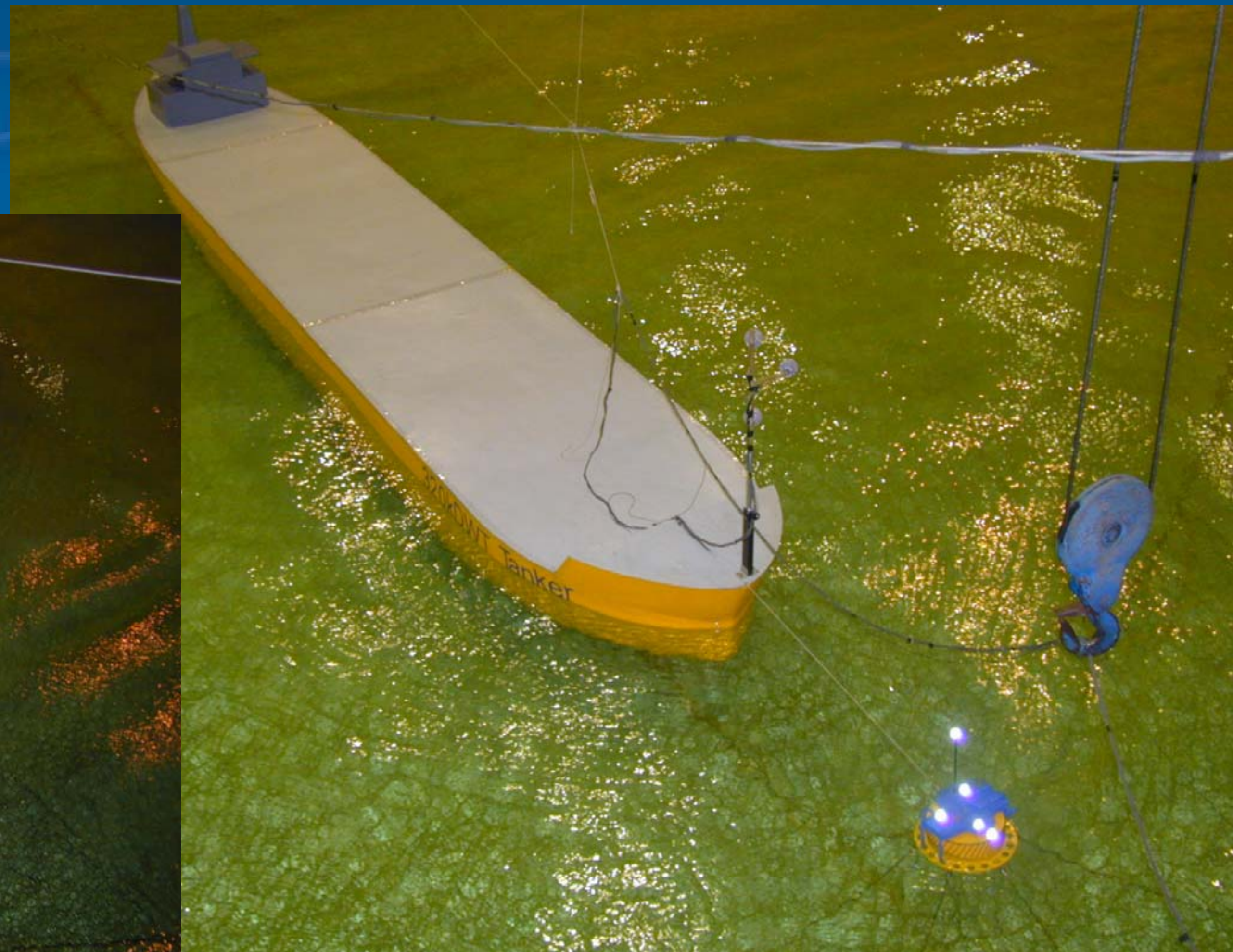
Summary

1. Freq-domain analysis methodology is presented as a tool of motion estimate even including LF components.
2. Classical linearization methods applied to quadratic drag term
3. Case studies carried out with a spread moored FPSO
4. Comparison b/w FD and TD calculations shows that the results are in good agreement.
5. Second-order wave potential is not significant for the relatively low frequency range.
6. Unstable time-varying yaw motion can only be analyzed by using a TD analysis.
7. Comments on modeling presented.

Future Work: Validation against Model Test Results



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Thank you very much!