Empirical Estimation of Probability Distribution of Extreme Responses of Turret Moored FPSOs

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Background

Design Value

Extreme Statistics

Extreme Value Analysis

Sample Data

Limited Data

Complexity of data structure

Distribution Model

Numerical

Experimental

Full Scale

Non-linearity

Mixed Effects
Table of Content

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Non-linear Responses of Turret Moored FPSOs

Mooring Leg Tension
- Windward
  - Low-Frequency
  - Wave Frequency
  - Total Tension
- Leeward
  - Low-Frequency
  - Wave Frequency
  - Total Tension

Vessel Offset
- Along the vessel length (X)
  - Low-Frequency
Sources of Non-linearity

- Mooring System Stiffness
- Loading Nature (low-drift forces, drag, etc.)
- Environmental Condition (steep waves)
- Damping
**Probability Distributions**

Normalized Random Variable \[ \zeta = \frac{a - \mu_{\eta}}{\sigma_{\eta}} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Transformation</th>
<th>Distribution</th>
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<tbody>
<tr>
<td><strong>Linear Random Variable</strong> (narrow-banded)</td>
<td>( F_{\zeta_n}(x) = 1 - \exp\left(-\frac{x^2}{2}\right) )</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>( \zeta_n = \frac{\xi^2}{2} )</td>
<td>( F_{\xi_n}(x) = 1 - \exp(-x) )</td>
<td>Exponential</td>
</tr>
<tr>
<td>( \zeta_n = A\left(\frac{\xi^2}{2} + B\right) )</td>
<td>( F_{\xi_n}(x) = 1 - \exp\left(-\left(\frac{x}{A} - B\right)\right) )</td>
<td>Stansberg</td>
</tr>
<tr>
<td><strong>Non-Linear Random Variable</strong></td>
<td>( F_{\zeta_n}(x) = 1 - \exp\left(-\left(\frac{x - \rho}{\lambda}\right)^\kappa\right) )</td>
<td>Weibull</td>
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<td>( \zeta_n = \frac{\lambda}{2^{1/\kappa}}\xi^{2/\kappa} + \rho )</td>
<td>( F_{\zeta_n}(x) = 1 - \exp\left(-\left(\frac{x - \rho}{\lambda}\right)^\kappa\right) )</td>
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<tr>
<td>( \zeta_n = \alpha \zeta + \beta \xi^2 + \gamma )</td>
<td>( F_{\xi_n}(x) = 1 - \exp\left(-\frac{(\chi - \alpha)^2}{8\beta^2}\right) )</td>
<td>3-Par Rayleigh</td>
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\[ \chi = \left(\alpha^2 + 4\beta(x - \gamma)\right)^{1/2} \]
## Distribution Parameters

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<td>Rayleigh</td>
<td>$\mu_\eta, \sigma_\eta$</td>
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<td>(narrow-banded)</td>
<td>Exponential</td>
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<td>Non-Linear Random Variable</td>
<td>Stansberg</td>
<td>$\mu_\eta, \sigma_\eta, A, B$</td>
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<tr>
<td></td>
<td>Weibull</td>
<td>$\mu_\eta, \sigma_\eta, \kappa, \rho, \lambda$</td>
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Extreme Statistics

Ordered Value Statistics Theory
(N independent cycles):

Expected Maximum:

Asymptotic Distribution of Large N (Gumbel)

Number of Cycles (N)

Wave frequency  Narrow-banded process  \[ N = \frac{T_{storm}}{T_z} \]

Low frequency  Non-narrow-banded– Correlation time - Stansberg’s formula  \[ \tau = \frac{1}{2\omega} \]

\( \omega \)  bandwidth of the spectrum

Combined Process  Difficult to estimate  Number of observed cycles
Case Studies: General Info

Deepwater System

Water depth (m) ~2000m
Area West Africa
100Yr Condition Hs = 4.5m, Tp = 17sec, Ws = 6.3m/sec
Mooring System 3G*4L Taut mooring legs
Mooring Legs Chain-Polyester-Chain

Shallow-water System

Water depth (m) ~45m
Area South East Asia
100Yr Condition Hs = 10m, Tp = 16sec, Ws = 32m/sec
Mooring System 4G*3L Catenary mooring legs
Mooring Legs Chain-Heavy Chain-Chain
Case Studies: Response Characteristics

Deepwater System

Shallow-water System
Case Studies: Response Characteristics

![Graph showing response characteristics for shallow water and deepwater](image)

- **X/\(\mu_X\)**
- **\(F_X/\mu_FX\)**

Legend:
- Shallow Water
- Deepwater
Results: Wave-Frequency

Deepwater windward

Deepwater leeward

Shallow-water windward

Shallow-water leeward

\[ P = (1 - u) \]
Results: Low-Frequency

Deepwater windward

Shallow-water windward

Deepwater leeward

Shallow-water leeward

\[ P = (1 - u) \]
Results: Total

Deepwater windward

Deepwater leeward

Shallow-water windward

Shallow-water leeward
## Results: Extreme Statistics

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<td>Sample</td>
<td>4.2</td>
<td>3.2</td>
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<td>(5.4 - 3.3)</td>
<td>(3.5 - 2.8)</td>
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<td>7.1</td>
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<td>3-Par. Weibull</td>
<td>3.9</td>
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Concluding Remarks

- The probability distribution of mooring leg tension and vessel offset in extreme environmental condition were studied.
- Two case studies of shallow water and deepwater turret moored FPSOs are considered.
- The characteristics of probability distribution of wave-frequency, low-frequency, and the combined tension are studied.
- The probability distributions of tension in the windward and leeward lines are studied.
- The performance of widely used distribution models as well as the three-parameter Rayleigh distribution model is evaluated over the experimental data.
- The effect of distribution model on the predicted extreme values is discussed.
Thank You!